D.K.M. COLLEGE FOR WOMEN(AUTONOMOUS),VELLORE-1.

DEPARTMENT OF MATHEMATICS

III B.SC MATHEMATICS

FUZZY MATHEMATICS

SUBJECT CODE: 11SMA6B

UNIT I **SECTION-A**

1.Define Fuzzy set with example.

2. Define crisp set.

3. Define subset with example.

4. Define height of fuzzy set.

5. Define subnormal fuzzy subset.

6. Define partial order relation.

7. Define partial ordered set.

8. Define Total ordered set.

9. Define upper and lower bound for fuzzy set.

10. Define Least upper bound (or) supremum.

11. Define greatest lower bound (or) Infimum.

12. Define Lattice.

13. Define Boolean algebra.

14. Define Union of fuzzy subsets.

15. Define Intersection of fuzzy subsets.

16. Define complementation with examples.

17. Define fuzzy subset function.

UNIT II

18.Define fuzzy relation with example.

19. Define inverse fuzzy relation with example.

20. Define height of fuzzy relation with relation.

21. Define support of fuzzy relation with example. with example.

22. Define algebraic of fuzzy relation with example.

23. Define crisp relation with example.

24. Define fuzzy compatability relation.

25. Define fuzzy partial ordered set with example.

26. Define dominating class of x with example.

27. Define fuzzy lattice.

28. Define fuzzy logic.

29. Define connectives with example.

30. Define conjunction with example.

31. Define Disjunction with example.

32. Define Truth table.

33. Define conditional statement with example.

34. Define biconditional statement with example.

35. Define contrapositive with example.

36. Define atomic statement with example.

37. Define molecular statement with example.

38. Define Tautology with example.

39. Define duality law with example.

UNIT III

40. Define fuzzy groupoids.

41. Define characteristic function.

42. Define lattice of fuzzy subgroup.

43. Define homomorphis of groupoids.

44. Define fuzzy invariant subgroup.

45. Define fuzzy quotient group.

UNIT IV

46. Define fuzzy subring with example.

47. Define fuzzy field with example.

48. Define fuzzy linear space with example.

UNIT V

49. Define fuzzy subspace with example.

50. Define Ideal with example.

51. Define right ideal with example.

52. Define left ideal with example.

SECTION-B

53.Write the properties of lattices.

54.Given $x \leq y \rightleftarrows (xVy) = x$. Prove that '$\leq$' is partial ordering.

55.Prove that $x \wedge y$ is the infimum of x and y.

56. Write the properties of fuzzy subset set.

57.Prove that $(\mu_1 \cup \mu_2) \cap \mu_3 = (\mu_1 \cap \mu_3) \cup (\mu_2 \cap \mu_3)$

58.Prove that $(\mu_1 \cap \mu_3)^c = \mu_1^c \cup \mu_2^c$

59.Write down the properties are satisfied by addition and product of fuzzy subsets.

60. Prove that (i) $(\mu_1 . \mu_3)^c = \mu_1^c + \mu_2^c$ (ii) $(\mu_1 + \mu_3)^c = \mu_1^c . \mu_2^c$

61.Let f is a function f: X→Y be a function then

(i) $f^{-1}[\eta^c] = (f^{-1}[\eta])^c$ for any subset η of Y.

(ii)$f[\mu^c] \supseteq (f[\mu])^c$ for any fuzzy subset μ of X if f is onto.

(iii)$\mu_1 \subset \mu_2 \Rightarrow f[\mu_1] \subset f[\mu_2]$ for any fuzzy subsets μ_1, μ_2 of X.

63. onstruct the truth table for $P \Rightarrow Q$, $Q \Rightarrow P$, $\neg P \Rightarrow \neg Q$, $\neg Q \Rightarrow \neg P$.

64. Construct the truth table for $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$.

65. Prove that $\{\uparrow\}$ is functionally complete set. To prove this it is sufficient to show that $\{\wedge, \neg\}$ and $\{\vee, \neg\}$ can be expressed in terms of $\uparrow$ alone.

66. For any $\theta \in [0,1]$, the θ level set $[\mu]_\theta = \{x \in S / \mu(x) \geq \theta\}$ is a subgroupoid if μ is a fuzzy subgroupoid.

67. Show that the intersection of any set of fuzzy subgroupoids is a fuzzy groupoids.

68. Show that $\mu(xy^{-1}) = \mu(e)$ implies $\mu(x) = \mu(y)$.

69. Every level subgroup $\mu_k(\theta < t \leq 1)$ of a fuzzy normal subgroup μ of a group G is a normal subgroup of G.

70. Every normal subgroup H of a group G can be considered as a level set of a fuzzy normal subgroup.

71. Let z be an ideal of a ring R if μ is a fuzzy left (right) ideal of R then the fuzzy subset η of $R/_I$ defined by $\eta(a+z) = \sup_{u \notin I} \mu(a+u)$ is a fuzzy left (right) ideal of the quotient ring $R/_I$.

72. F is a fuzzy field X iff, (i) $\mu_F(x-y) \geq \min(\mu_F(x), \mu_F(y))$ for all $x, y \in X$.

(ii) $\mu_F(xy^{-1}) \geq \min(\mu_F(x), \mu_F(y))$ for all $x \in X, y \neq 0 \in Y$.

73. Let R be skew field and ($\mu \neq 0$) be a fuzzy subset of R, then μ is a fuzzy (left, right) ideal of R iff $\mu(x) = \mu(e) \leq \mu(0)$ $\forall x(\neq 0) \in R$ where 0 is the unity in $(R,+)$ and e is the unity of $(R,\bullet)$.

74. Let X and Y be fields and f is a homomorphism from X into Y. Let F be a fuzzy field in Y. Then the inverse images $f^{-1}(F)$ of f is a fuzzy field in X.

75. Let V be a fuzzy linear space in Y (over the fuzzy field F in X) if $\mu_v(\lambda x + \mu y) \geq \min\{\min\{\mu_F(\lambda), \mu_v(x)\}, \min\{\mu_F(\mu), \mu_v(y)\}\}$ $\forall \lambda, \mu \in X$ & $x, y \in Y$ then $\lambda_v(\lambda x + \lambda y) \geq \min(\mu_v(x), \mu_v(y)) \forall x, y \in Y$.

76. If L is a complete lattice then the intersection of a family of fuzzy linear space is a fuzzy linear space.

UNIT-V

77. Let μ be a fuzzy subset of a vector space X then the following are equivalent:

(i) μ is a fuzzy subspace of X.

(ii) $a\mu + b\mu \subset \mu$ For all scalars a and b.

(iii) $\mu(ax+b) \geq \min\{\mu(x), \mu(y)\}$ For all scalars a and b and for all x, y in X.

78. Let X and Y be linear space over the same field and $f : X \to Y$ be linear map

(i) If μ is a linear subspace of X, then f(μ) is a linear subspace of Y.

(ii) If η is a fuzzy subspace of Y then $f^{-1}(\eta)$ is a fuzzy subspace of X.

79. If μ, μ_1 and μ_2 are fuzzy subspaces of a linear space X then $\mu_1 + \mu_2$ and kμ are also fuzzy subspace of X for any scalar k.

80. If $\{\mu_i\}$ be a family of linear subspaces of a linear space X. Then $\bigcap_i \mu_i$ are also fuzzy subspaces of X.

81. For a fuzzy subset μ of the linear space X the following are equivalent

(i) μ is affine (convex).

(ii) $\mu(kx + (1-k)y) \geq \min(\mu(x), \mu(y))$.

(iii) $[\mu]_\alpha = \mu^{-1}[\alpha, 1]$ is ordinary affine (convex).

82. For a fuzzy subset μ of the linear space X the following are equivalent

(i) μ is balanced.

(ii) $\mu(xx) \geq \mu(x) \forall x \in X$ and all scalar k with $|k| \leq 1$.

(iii) $[\mu]_\alpha = \mu^{-1}[\alpha, 1]$ is balanced.

83. X and Y are two linear space and f: $X \to Y$ is a linear map if μ is affine (convex/balanced) fuzzy subset of X then f(μ) is affine (convex/balanced) fuzzy subset of Y.

84. If μ_1 and μ_2 are affine (convex/balanced) fuzzy subset of X then so is $\mu_1 + \mu_2$.

85. Let A is a fuzzy algebra in Y over F iff for all x, y$\in Y$ and λ_1, $\lambda_2 \in X$

(i) $\mu_A(\lambda_1 x + \lambda_2 y) \geq \min\{\min\{\mu_F(\lambda_1), \mu_A(x)\}, \min\{\mu_F(\lambda_2), \mu_A(y)\}\}$

(ii) $\mu_A(xy) = \min\{\mu_A(x), \mu_A(y)\}$