**D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1.**

**TRIGNOMETRY**

**UNIT- I SECTION-A (2 MARKS)**

1. Write the expansion for sinn$\theta$ *and* $cosn\theta$.
2. Write the expansion for tann$\theta$.
3. Expand tan7$\theta$ in terms of tan$\theta$.
4. What is the expansion of sin5$\theta$.
5. Find the three roots of the equation $ax^3+bx^2+cx+d=0$.
6. Write the formula for tan($\theta 1+\theta 2+\cdots+\theta n$).
7. Write the formula for sinA+sinB, cosA-cosB.

**SECTION-B (5 MARKS)**

1. Expand cos4$\theta$ in terms of sin$\theta$.
2. Expand sin7$\theta$
3. If $\alpha,\beta,\gamma$ are the roots of the equation $x^3+px^2+qx+p=0$ prove that $\tan^{-1}\alpha+\tan^{-1}\beta+\tan^{-1}\gamma=n\pi$ of radians.
4. If tan$\theta$1,tan$\theta$2,tan$\theta$3 are the roots of the equation $x^4+px^3+qx^2+rx+s=0$ then show that tan($\theta 1+\theta 2+\theta 3+\theta 4$)=r-p/1-q+s.
5. Solve the equation $\sin 7\theta-sin\theta=sin3\theta$.
6. Solve the equation $sin5\theta+sin\theta=sin3\theta$.
7. Solve the equation $tan3\theta-4tan\theta=0$.

**SECTION-C(10 MARKS)**

1. Prove that $cos8\theta=1-32sin^2\theta+160\sin^4\theta-256\sin^6\theta+128sin^8\theta$.
2. Prove that $\sin 5\theta/\sin\theta=5-20\sin^2\theta+16\sin^4\theta$.
3. Prove that the equations $ahsec\theta-bkcosec\theta=a^2-b^2$ has 4 roots and that the sum of the values of $\theta$ satisfy it and it is equal to an odd multiples of $\pi$ radians.
4. Solve the equation 4sinx+3cosx=1

**UNIT II SECTION A (2 MARKS)**

1. Derive the formula for $x^n+1/x^n$ & $x^n-1/x^n$.
2. Write the expansion of sin$\theta$ *and* $cos\theta$ in ascending powers.
3. Expand tan$\theta$ in ascending power.
4. Prove that 1)$\text{cosec}^{-1}x=\sin^{-1}(1/x)$, 2)$\cot^{-1}x=\tan^{-1}(1/x)$.
5. Define inverse circular function.
6. Prove that $\sin^{-1}(x)+\cos^{-1}(x)=\pi/2$.
7. Prove that $\tan^{-1}x+\cot^{-1}x=\pi/2$.
8. Prove that $2\tan^{-1}x=\tan^{-1}(2x/1-x^2)$.
9. Solve $\tan^{-1}(2x)+\tan^{-1}(3x)=3\pi/4$.

**SECTION B (5MARKS)**

1. Show that $\sin^{-1}(3/5)+\sin^{-1}(8/17)=\sin^{-1}(77/85)$.
2. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi/2$ show that xy+yz+zx=1.
3. Prove that $\sin^{-1}x-\sin^{-1}y=\sin^{-1}[x\sqrt{1-y^2}-y\sqrt{1-x^2}]$.
4. If $\sin\theta/\theta=5765/5766$ then find $\theta$ approximately.

5. Determine it a,b if $\lim\theta \to_0$ $(a-\theta sin\theta - bcos\theta)/\theta^4=1/12$.
6. Evaluate $\lim_{x\to 0}$ $(sinx-xcosx)/x^3$.
7. Evaluate $\lim_{x\to 0}$ $(tan2x-2tanx)/x^3$.
8. Prove that $16cos^5\theta = cos5\theta + 5cos3\theta + 10cos\theta$.
9. Prove that $32cos^6\theta = cos6\theta + 6cos4\theta + 15cos2\theta + 10$.

**SECTION C (10 MARKS)**

1. Expand $sin^8\theta$.
2. Show that $128sin^8\theta = cos8\theta - 8cos6\theta + 28cos4\theta - 56cos2\theta + 35$.
3. Prove that $32sin^4\theta cos^2\theta=cos6\theta - 2cos4\theta - cos2\theta + 2$.
4. Prove that $64(cos^8\theta + sin^8\theta) = cos8\theta + 28cos4\theta + 35$.
5. Evaluate lt x→0 (tanx-sinx)/sin3x.
6. Show that $\lim_{x\to \frac{\pi}{2}}(cos2x + sinx)/cos^2x=3/2$.
7. Determine a,b,c show that $\lim\theta \to 0$ $(\theta(a + bcos\theta) - csin\theta)/\theta^5=1$.
8. Show that $tan^{-1}x+tan^{-1}y+tan^{-1}z=tan^{-1}[(x+y+z-xyz)/1-xy-yz-zx]$.

**UNIT III SECTION A (2 MARKS)**

1. Write the expansion of $cos\theta, sin\theta$ in terms of $\theta$.
2. Prove that $cosh^2x-sinh^2x=1$.
3. Prove that 2sinhx coshx=sin2hx.
4. Prove that $cosh2x=cosh^2x+sinh^2x$.
5. Prove that $1-tanh^2x=sech^2x$.
6. Solve $cosec^2\theta = 1 + cot^2\theta$ in terms of hyperbolic function.
7. Prove that $sinh3x=3sinhx+4sinh^3x$.
8. Prove that $cosh3x=4cosh^3x-3coshx$

**SECTION B (5 MARKS)**

1. Prove that $tanh^{-1}x=1/2log_e(1+x/1-x)$.
2. Prove that $cosh^{-1}x=log_e(x+\sqrt{x^2-1})$.
3. Prove that $sinh^{-1}x=log_e(x+\sqrt{x^2+1})$.
4. Show that $tanh3x=tanh^3x+3tanhx/1+3tanh^2x$.
5. If cosh(a+ib) cos(c+id)=1 prove that (1)cosb cosc cosha coshd+sinb sinc sinha sinhd=1 (2)tanha tanb=tanhd tanc.
6. Find the value of tanh(x-y).
7. Find the value of tanh(x+y).
8. Prove that sinh(x+y)=sinhx coshy+coshx sinhy.
9. Prove that $cosh^2x+sinh^2x=cosh2x$.

**SECTION C (10 MARKS)**

1. Prove that sinh(x-y)=sinhx coshy-coshx sinhy.
2. If $cos\alpha\ cosh\beta = cos\varphi, sin\alpha\ sinh\beta = sin\varphi$ prove that $sin\varphi = \pm sin^2\alpha = \pm sinh^2\beta$.
3. If $cos(x+iy)=cos\theta + isin\theta$ prove that $cos2\theta + cosh2y = 2$.
4. If sin(A+iB)=x+iy prove that 1)$(x^2/sin^2A)-(y^2/cos^2A)=1$ , 2)$(x^2/cosh^2B)+(y^2/sinh^2B)=1$
5. If tan(x+iy)=U+iV than prove that U/V=sin2x/sin2hy.
6. Separate into real and imaginary parts of $tan^{-1}(x+iy)$.
7. If coshu=sec$\theta$ show that $u=log[tan(\frac{\pi}{4} + \theta/2)]$

## UNIT IV SECTION A (2 MARKS)

1. General value of logirthemic of (x+iy).
2. Find log(1-i)
3. Find log(4+3i)
4. Find log(1+i)
5. Find log(3+4i)

## SECTION B (5 MARKS)

1. Show that log(1+itan$\alpha$) = log($sec\alpha$) + $i\alpha$.
2. Prove that log(a+ib)/(a-ib)=2itan$^{-1}$(b/a).
3. Find the value of $i^i$.
4. Sum to infinity of the series $\sin^3(\theta/3)+3\sin^3(\theta/3^2)+3^2\sin^3(\theta/3^3)+\ldots\infty$
5. Find the sum to nterms tan$\alpha$ + $2tan2\alpha + 2^2\tan(2\alpha)^2+..\infty$
6. Find the sum to series $\cos^2x+\cos^2(x+y)+\cos^2(x+2y)+..$ upto n series.

## SECTION C (10 MARKS)

1. Sum to n terms and if possible to infinity $\tan^{-1}(1/1+1+1^2)+\tan^{-1}(1/1+2+2^2)+\tan^{-1}(1/1+3+3^2)+\ldots$
2. Sum to infinity the series c=cos$\alpha + \frac{1}{2}\cos(\alpha+\beta)$ +(1/2)(3/4)cos$(\alpha + 2\beta)$+..
3. Find the value of 1-(1/2)cos$\theta$ +(1.3/2.4)cos2$\theta - \frac{1.3.5}{2.4.6}cos3\theta+..\infty$
4. Find the sum of the series sinhx+sinh(x+y)+sinh(x+2y)+…to n terms.
5. The sum of the series sines of angles in A.P.
6. Find the sum to 'n' terms of the series $1/(\cos\theta - cos3\theta)$ + $1/(\cos\theta - cos5\theta)$ +$1/(\cos\theta - cos7\theta) + \cdots$
7. Find the sum to n terms of the series $\cos ec\theta.cosec2\theta + cosec2\theta\ cosec3\theta+..\infty$
8. If $i^{ii\cdots\infty} = A + iB$ then prove that tan$A\frac{\pi}{2} = B/A$ and $A^2+B^2=e$ .
9. Show that $\log_1 i$=4n+1/4m+1 where m,n are integer.
10. Show that log tan$(\frac{\pi}{4} + i(\frac{x}{2}))$=itan$^{-1}$(sinhx).
11. If $i^{x+iy}$=A+iB show that $A^2+B^2=e^{-(4n+1)}\pi y$.
12. If log sin$(\theta + i\varphi) = A + iB\ show\ that$ 1)$2e^{2A}$=cosh2$\varphi - cos2\theta$ 2) $\cos(\theta - B) = e^2\varphi\cos(\theta + B)$
13. Find the value of log((1+cos$\theta + isin\theta)/cos\theta - 1 + isin\theta$)
14. Find the logrithemic of x+iy.

## UNIT V SECTION A (2 MARKS)

1. Find the expansion of $\tan^{-1}$x.
2. Find the value of $\pi\ using\ tan^{-1}$x expansion.

## SECTION B (5 MARKS)

1. If $\theta$ lies between $\pm\frac{\pi}{4}\ then\ \theta = tan\theta - tan^3\frac{\theta}{3}$ +$\tan^5\frac{\theta}{5}$..
2. Prove that $\tan^{-1}(1/2)+\tan^{-1}(1/3)=\pi/4$.
3. Prove that $\pi = 2\sqrt{3}(1-(1/3^2)+(1/5)(1/3^2)-(1/7)(1/3^3)+..)$
4. Prove that $\frac{\pi}{12}$ =$(1-1/3^{1/2})-(1/3)(1-1/3^{3/2})+(1/5)(1-1/3^{5/2})\ldots\ldots\infty$

## SECTION C (10 MARKS)

1. Prove that $1+(1/3)-(1/5)-(1/7)+..=\pi/2\sqrt{2}$.
2. Derive Gregory's series, Euler's series expansion.