D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1.

ALGEBRA-II

UNIT-1 SECTION-A 6 MARKS

1.State& Prove Transitivity of a finite extension.

2.If L is a finite extension of F and if K is a subfield of L which contains F then [K:F]/[L:F].

3.Every finite extension is a algebraic extension.

4.If a∈K is algebraic of degree n over F.Then [F(a):F]=n

5.If a,b in K are algebraic over F.Then $a\pm b$, ab,a/b if b≠0 are all algebraic over F. In otherwords the element K which are algebraic over F form a subfield of K.

6.If L is algebraic extensions of K and K is a algebraic extension of F. Then show that L is a algebraicextension of F.

7.Let F be a field and f(x) be a ring of polynomial over F.Let g(x) be a polynomial of degree n in F[x]. The ring of all polynomial V=g(x) be the ideal generated by g(x), g∈F[x] prove that $\frac{F[x]}{V}$ is adimensional vector space over F.

8.If a∈K is algebraic over F and P(x) is irreducible polynomial of degree n over F. Show that F(a) is afinite extension of F.

9.Let R be a field of real number and Q be a field of rational number. Show that $\sqrt{2}$&$\sqrt{3}$ are

algebraic over Q and exist a polynomial of degree 4 over Q satisfies by $\sqrt{2}$ +$\sqrt{3}$.

10.Prove that $\frac{d^i}{dx^i}\frac{g(x)}{p-1!}$ is divisible by P where g(x)=$\sum_{n=1}^{k} a_n\, x^n$, i$\geq P$.

SECTION-B 15 MARK QUESTIONS:

11.The element a∈K is algebraic over F iff F(a) is a finite extension of F.

12.Prove that e is Transcendental.

UNIT-II SECTION-A 6 MARK QUESTIONS

1.State & prove Remainder Theorem.

2.State & prove Factor Theorem.

3.Let a∈K be the root of p(x)∈F[x] of multiplicity m and if p(x)=$(x-a)^m$ q(x) then any other root of

P(x) in K must be a root of q(x)∈K[x] in the field K. Conversely any other root of q(x) is also a root of P(x).

4.If p(x) is a polynomial in F[x] of degree n≥ 1 and it is irreducible over F. then there is an extension

E of F such that [E:F]=n in which p(x) has a root in E.

5.Ʈ* defines an isomorphism of F[x] onto F'[t] with the property that α(Ʈ*)=α' for α∈F.

6.If p(x)∈F[x] is irreducible and if a,b are the roots of p(x).Then F(a)≅F(b) by an isomorphism which

takes a$\xrightarrow{onto} b$ and leaves every element of F fixed.

7.For every f(x),g(x)∈F[x] for every α∈F. Prove that i)(f(x)+g(x))'=f'(x)+g'(x) ii)(αf(x))'=α f'(x)

iii)(f(x)g(x))'=f(x)g'(x)+f'(x)g(x).

8.The Polynomial f(x)∈F[x] has a multiple root iff f(x) and f'(x) have a non-trivial common factors.

9.If f(x)∈F[x] is irreducible. Then

i)char f=0 then f(x) has no multiple roots

ii)If char f=p≠0 then f(x) is multiple roots if it is of the form f(x)=g(x^p).

10.If F is a field of Char f=p≠0 then the polynomial x^{p^n}-x∈F[x] where n≥1 has distinct roots.

11.show that any field of character zero is perfect.

12.If a,b are seperable over F of charF=0 then prove that F(a,b) is a simple extension.

13.In particular any 2 splitting field of the same polynomial over a given field F are isomorphism by

an isomorphism leaving for all element of F fixed.

14.If p is a prime nmber the splitting field over F the field of rational number of the polynomial x^p-1

Is of degree p-1.

15.If E is an extension of F and f(x)ЄF[x] and ф is an automorphism of E leaving element of F fixed.

Prove that ф must take a root of f(x) lying in E into a root of f(x) in E.

16.Prove that if the complex number Z is a root of a polynomial p(x) having real co-efficients then $\bar{z}$

the complex conjugate of z is also a root of p(x).

17.Prove that m is an integer which is not a perfect square and if α+β($\sqrt{m}$), [α,β rational] is the root

of a polynomial .p(x) having rational co-efficient, then α.β$\sqrt{m}$ is also a root of p(x).

SECTION-B 15 MARK QUESTIONS

18.If f(x)ЄF[x] then there is a finite extension E of F in which f(x) has a root in E. Moreover

[E:F]≤degf(x).

19.Let f(x)ЄF[x] be of degree n≥1 then prove that there is an extension E of F degree atmost factorial

Of n in which f(x) has n roots.

20.Prove that a polynomial of degree n over a field F[x] can be atmost n roots in any extension field.

21.If p(x) is irreducible polynomial in f[x] and if V is a root of p(x) . then F(V) is isomorphic to F'(w)

where w is the root of p'(t). Moreover the isomorphism σ can be choosen that i)V(σ)=w

ii)α(σ)=α' for all αЄF.

22.If F is of char0 and if a and b are algebraic over F.Then there exists an elements cЄF[a,b] such that

F[a,b]=F[c].

23.Any finite extension of a field of char0 is a simple extension.

24.Any splitting field E and E' of the polynomial f(x)ЄF[x] and f'(t)ЄF'[t] respectively are isomorphic by an isomorphism ф with the property αф=α' for all αЄF.

UNIT-III SECTION-A 6 MARK QUESTIONS

1.If K is a field and if σ_1,σ_2 are distinct automorphism of K then it is impossible to find the elements

a_1,a_2,.......a_n not all zero in K such that $a_1\sigma_1$(u)+$a_2\sigma_2$(u)+..........+$a_n\sigma_n$(u)=0.

2.Fixed field of G is a subfield of K.

3.Show that G(K,F) is a subgroup of automorphism of G.

4.If K is a finite extension of F then G(K,F) is a finite group and its order of G(K,F) satisfies the

Inequality O(G(K,F)) ≤[K:F].

5.Let K be a normal extension of a field F of charF=0. Then [K:F]=O(G(K,F)).

6.Let K be the splitting field of f(x) in F[x]. Let p(x) be an irreducible factor r of f(x) in F[x]. If the root

of p(x) are α_1,α_2,........α_r then for each i there exist an automorphismσ_i in G(K,F). Such that

$\sigma_i(\alpha_1)$=α_i.

7.Let f(x)ЄF[x] be an irreducible polynomial and charF=0. Then f(x) has no multiple roots.

SECTION-B 15 MARK QUESTIONS

8.Let K be a normal extension of F and charF=0. If T is the subfield of K containing F. Then T is the

Normal extension of F⇔σ(T)ϹT.

9.state & prove Fundamental theorem of Galoi's Group.

10.Let F be a field and F(x_1,x_2,....x_n) be the field of rational function in x_1,x_2,.......x_n over F. Suppose S is the field of symmetric rational function

i)F(x_1,x_2,......x_n) over n! i.e)[F(x_1,x_2,......x_n):S]=n!.

ii)G((x_1,x_2,....x_n),S)=S_n where S_n is a symmetric group of degree n.

iii)S=F(a_1,a_2,.....a_n) if a_1,a_2,.....a_n has elementary symmetric functions of x_1,x_2,......x_n. Iv)F(x_1,x_2,....x_n) Is the splitting field over F(a_1,a_2,....a_n)=S of the polynomialt^n-a_1t^{n-1}+a_2t^{n-2}....(-1)ⁿa^n.

11.Suppose K is a finite extension of F char 0 and H is a subgroup of G(K,F). Let K_H is a

Fixed field of H. Then i)[K:K_H]=O(H) ii)H=G(K,K_H).

12.If K is a normal extension of F iff K is the splitting field of some polynomial over F.

UNIT-IV SECTION-A 6 MARK QUESTIONS

1.Let F be a finite field having q elements. Let F⊂K where K is a finite field & [K:F]=n then K has

q^n elements.

2.Let F be a finite field then F has p^m elements where the prime number p is the charF

i.e)charF=P.

3.If the finite field F has p^m elements then for all a∈F, satisfies a^{p^m}=a.

4.If the finite field F has p^m elements then the polynomial x^{p^m}-x in F[x] can be

Factorized as x^{p^m}-x=$\prod_{\lambda \in F}(x-\lambda)$.

5.If the field F has p^m elements then F is the splitting field of the polynomial x^{p^m}-x In F[x].

6.Any 2 finite fields having the same number of elements are isomorphic.

7.For every prime p & every positive integer m then there exist a field having p^m elements.

8.If F is a finite field and α≠0,β≠0 are 2 elements of F then we can find a&b in F

Suchthat $1+\alpha a^2+\beta b^2=0$.

9.Let G be a finite abelian group then for every integer n, the relation x^n=e is satisfied

by atmost n elements of finite abelian group G. Prove that G is a cyclic group.

10.Let K be the field & G be finite subgroup of the multiplication group of non-zero

Elements of K then G is cyclic group.

11.The multiplicative group of non-zero elements of a finite field is cyclic.

SECTION-B 15 MARK QUESTIONS

1.State and prove Wedder Burns theorem.

UNIT-V SECTION-A 6 MARK QUESTIONS

1.Let G' be a commutator subgroup of G then G is abelian⇔G'= $\{e\}$.

2.Let G'be a commutator subgroup of G then i)G' is normal in G. ii)$^{G}/_{G'}$ is abelian.

3.Let G' be the commutator subgroup of G then G' is generated by U where

U={$x^{-1}y^{-1}$xy/x,y∈G}. Let H be a normal subgroup of G then $\frac{G}{H}$ is abelian⇔G'$\subseteq$H.

4.The adjoint in Q satisfies the following i)x^{**} =x ii)(δx+ϑy)*=δx*+ϑy* iii)(xy)*=y*x*.

5.If for all x,y∈Q& N(xy)=N(x)N(y).

6.State and prove Lagrange's Identity.

7.H is a subring of Q, if x∈H then x*∈H & N(x) is a positive integer for every non-zero x in H.

8.State and prove Left Division Algorithm.

9.Let L be the left sided ideal of H then there exists an element u∈L such that x=cu for

every x∈L, where c∈H.

10.If a∈H then a^{-1}∈H iff N(a)=1.

11.Let c be the field of complex numbers and suppose that the division ring D is algebraic Over C. Then D=C.

SECTION-B 15 MARK QUESTIONS

12. State and prove Four square theorem.

13. State and Prove Theorem of Frobenius.