D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1.

COMPLEX ANALYSIS

UNIT I SECTION-A 2 Marks

1. Define Triangle inequality.

2. Prove that $|z-1| = |z+i|$ represents the line through the origin

   Origin whose slope is -1

3. The limit of a function is unique.

4. Define entire function.

5. Define harmonic function.

6. Prove that $u=y^3-3x^2y$ is harmonic

7. If $f'(z)=0$ in a domain D. then f(z) must be a constant throughout D

8. Define analytic function.

9. Define continuity of a function

10. Define derivatives of a function

11. Prove that $f(z)=z^2$ is harmonic

12. If v=2xy. Find f(z)

SECTION-B 5 Marks

*1.*Supposethat$f(z)=u(x,y)+iv(x,y)$,$z_0=x_0+iy_0$,$w_0=u_0+iv_0$then
$\lim_{z\to z_0} f(z)=w_0$ if $\lim_{(x,y)\to(x_0,y_0)} u(x,y)=u_0,\ \lim_{(x,y)\to(x_0,y_0)} u(x,y)=v_0$

2. Explain stereographic projection.

3. If the derivatives are f and F exists at a points Z then

   i. $(f \pm F)'(z)=f'(z) \pm F'(z)$
   ii. $(f \,.\, F)'(z)=f(z)F'(z)+f'(z)F(z)$

iii. $\left(\frac{f}{F}\right)'(z) = \frac{F(z)f'(z)-F'(z)f(z}{(F(z))^2}$

Provided $F(z)\neq 0$

4. Necessary condition for a function f to be differentiate at z_0.suppose that $f(z)=u(x,y)+iv(x,y)$ and $f'(z)$ exist at $z_0=x_0+iy_0$ then the first order partial derivatives of u and v must exists at(x_0,y_0) and they satisfy Cauchy-Riemann equations. $u_x=v_y$, $u_y=-v_x$ and $f'(z)$ can be written as $f'(z_0)=u_x+iv_x$. Where this partial derivatives are to be evaluated at(x_0, y_0).

5. For any function of $\boldsymbol{\phi}\frac{\partial^2\emptyset}{\partial x^2}+\frac{\partial^2 \emptyset}{\partial y^2}=\frac{4\,\partial^2\emptyset}{\partial z\,\partial\bar{z}}$

6. An analytic function with constant modulus is constant.

7. If $f(z)=u+iv$ is an analytic form of z and $u-v=\frac{\cos x+\sin x-e^{-y}}{2\cos x-e^{y}-e^{-y}}$ find f(z) subject to the condition $f\left(\frac{\pi}{2}\right)=0$

8. Prove that $\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}\right)\left|f(z)\right|^p = p\left|f(z)\right|^{p-2}\left|f^1(z)\right|^2$

9. If $u=e^x(x \cos y - y \sin c)$. Prove that u is harmonic and find $f(z)= u+iv$

SECTION-C 10 MARKS

1. If f is differentiable at z_0, then f is continuous at z_0
2. Sufficient condition for a function f(z) to be differentiable at z_0.
3. Let the function $f(z)=u(r,v)+iv(r,\boldsymbol{\theta})$. We define throughout some $\boldsymbol{\varepsilon}$-neighbourhood of a non-zero point $z_0=r_0\exp(iv_o)$
4. If f(z) is analytic function of z

   Prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) = 4\left(\frac{\partial^2 z}{\partial z\partial\bar{z}}\right)$

   Prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\left|f'(z)\right|^2$
5. Necessary condition for a function f to be differentiable at z_o.
6. State and prove Cauchy-Riemann equations in polar form.

UNIT II SECTION-A 2 MARKS

1. $\boldsymbol{\omega}$=b+Az where A&B are complex constant.
2. Find the image strip 0<x<1 under the transformation w=iz
3. Define isogonal and conformal transformation
4. Define critical point .with example
5. Define w=z^2
6. Define Bi-linear Transformation.
7. Define cross-Ratio.

SECTION-B 5 MARKS

1. Every bilinear transformation preserves the cross ratio
2. Find the bilinear transformation with maps z_1=-1,z_2=0,z_3=1 to w_1=-I,w_2=1,w_3=i
3. Every bilinear transformation which has only one fixed point αcan be put in the form $\frac{1}{w-\propto} = \frac{1}{z-\propto} + \lambda$
4. Find the fixed point and bring it to the normal form of following bi-linear transformation.$\omega = \frac{(z+i)z-2}{z+i}$
5. Find the bi-linear transformation which transformation the half plane R(z)≥0 into the circle $|\omega| \leq 1$
6. Find the mobius transformation which transformation the circle $|z| = 1$ and $|\omega| = 1$ and makes the points z=1,-1 correspondence to $\boldsymbol{\omega}$=1,-1

SECTION-C 10 MARKS

1. Discuss the transformation $\boldsymbol{\omega}=\frac{1}{z}$
2. Necessary condition for conformality.
3. Sufficient condition for conformality.
4. Discuss the transformation $\boldsymbol{\omega}=e^z$
5. Discuss the transformation $\boldsymbol{\omega}=z^2$
6. Find the bilinear transformation which transforms the circle
   $|z| \leq \rho$to the circle of $|\omega| \leq \rho$

UNIT III SECTION-A 2 MARKS

1. Define arc and simple arc.
2. Define simple closed curve and Jordan curve .
3. Find the value of integral I=$\int_c \bar{z}\,dz$ where c is the right hand half z=$2e^{i\theta}$ $\left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$ of the circle $|z| = 2$ from z=-2i to z=2i
4. Define multiple connected domain.
5. Define Cauchy-goursat for multiple connected domain.
6. Evaluate: $\int_c \frac{z.dz}{2z+1}$

SECTION-B 5 MARKS

1. C_1 denote the contour OAB find $\int_c f(z)dz$ where f(z)=y-x-i3x^2,O(0,0),A(i),B(1+i)
2. I=$\int_c z^{\frac{1}{2}}dz$ where c is the semicircle z=$3e^{i\theta}$ where 0≤$\boldsymbol{\theta}$≤**π** from the point z=3 to z=-3
3. Evaluate : $\int_c \frac{dz}{z(z-2)}$ where c is |z|=1
4. $\int_\Gamma \frac{e^{5z}}{z^3}dz$ where Γ́ is a circle |z|=1. Trace the anticlockwise direction
5. using Cauchy's integral formula, evaluate
6. $\int_c \frac{zdz}{(9-z^2)(z+i)}$ where c is the circle |z|=2
7. Find the value of the integral
8. $\int_0^{1+i} (x-y+ix)dz$ along the straight line from z=0 to z=1+i

SECTION-C 10 MARKS

1. State and prove cauchy's theorem.
2. If a function f is analytic at a point then its derivatives of all orders at that point and they are analytic.
3. State and prove maximum modulas theorem.
4. Suppose that f(z) is analytic throughout a neighbourhood $|z - z_0| < \epsilon$of a point z_0. If $|f(z)| \leq |f(z_0)|$ for each point z in that neighbourhood then f(z) has a constant value f(z_0) throughout the neighbourhood.

5. State and prove maximum Cauchy integral formula.

UNIT IV SECTION-A 2 MARKS

1. Define convergent and divergent series.
2. Define maclaurin series.
3. Define pole.
4. Define essential singular point and removable singular point
5. Define principal part of f at z_0.
6. Define convergence of the series
7. Prove absolute convergence implies convergence
8. A series converges iff the sequence of remainders tends to zero

SECTION-B 5 MARKS

1. Suppose that $z_n = x_n + iy_n$, n=1,2…,z=x+iy then $lim_{n\to\propto} z_n = z \Leftrightarrow lim_{n\to\propto} x_n = x ; lim_{n\to\propto} y_n = y$.
2. Suppose that $z_n = x_n + iy_n$, n=1,2…,z=x+iy then $\sum_{n=1}^{\propto} x_n = X \text{and} \sum_{n=1}^{\propto} y_n = y$
3. F(z)=$\frac{1}{z(z^2-3z+2)}$ for the region
   i. $0<|z|<1$
   ii. $1<|z|<1$
   iii. $|z|>1$
4. *Find the Laurent's expansion for f(z)=* $\frac{1}{(z+1)(z+3)}$ *in the domain* $1<|z|<3$.
5. *Find the Laurent's expansion for f(z)=* $\frac{z+3}{z(z^2-z-2)}$ *in the domain* $1<|z|<2$
6. *4. Evaluate* $\int_c \frac{3z-4}{z(z-1)(z+2)}dz$ *where C is* the *circle* $|z|=3/2$

SECTION-C 10 MARKS

1. State and prove Taylor's series.
2. State and prove Laurent's series.

UNIT V SECTION-A 2 MARKS

1. Find the poles and residues f(z)=$\frac{1}{z+z^2}$
2. Find the poles and residues f(z)=$\frac{1}{z^2+a^2}$
3. Define Residue
4. Find the Residue at Singularity f(z)=$\dfrac{z+1}{z^2-3z+2}$
5. Write the formula to compute the residue of f(z) at a pole of order m at z_0
6. State Cauchy's Residue theorem
7. Find the Residue and pole of z+$\dfrac{3}{z-2}$
8. Find the Residue and pole of f(z)= $\dfrac{z^3+3z}{(z-i)^3}$

SECTION-B 5 MARKS

1. State and prove Cauchy residue theorem.
2. An isolated singular point z_0 of a function f is a pole of order m iff f(z)=$\frac{\emptyset(z)}{(z-z_0)^m}$ where $\boldsymbol{\phi}$(z) is analytic an non-zero at z_0 and if m=1$res_{z=z_0}f(z) = \emptyset(z_0)$and if m≥ 2 $res_{z=z_0}f(z) = \frac{\emptyset^{m+(z_0)}}{m!-1}$
3. $\int_c \frac{z+1}{z^2-2z}dz$ where c is the circle $|z| = 3$
4. $find\ the\ pole\ and\ residue$ f(z)=$\left(\frac{z}{2z+1}\right)^3$
5. $find\ the\ pole\ and\ residue$ f(z)=$\frac{1}{z^2+a^2}$

SECTION-C 10 MARKS

1. Show that $\int_0^{\propto} \frac{x^2\,dx}{(x^2+1)(x^2+4)} = \frac{\pi}{6}$
2. Show that $\int_{-\propto}^{\propto} \frac{cos\ 3x\ dx}{(x^2+1)^2} = \frac{2\pi}{e^3}$
3. Prove that $\displaystyle\int_0^{\infty} \frac{1}{1+x^2}dx = \frac{\pi}{2}$

4. Prove that $\int_0^{\infty} \frac{1}{x^4+1} dx = \frac{\pi}{2\sqrt{2}}$

5. Evaluate $\int_0^{\infty} \frac{1}{(x^2+1)^2} dx$

6. Evaluate $\int_0^{\infty} \frac{\cos x}{1+x^2} dx$

7. Prove that $\int_{-\infty}^{\infty} \frac{\cos ax}{x^2+1} dx = \frac{\pi}{2} e^{-a}$