# D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1

## Question Bank

## COMPLEX ANALYSIS-I

**UNIT-I SECTION-A (6 MARKS)**

1. Prove that Cauchy's Integral Formula.
2. Prove that n(c,a)=1, compute $\int_{|Z|=1} \frac{e^Z}{Z}\, dZ$.
3. State and prove Morera's Theorem.
4. State and prove Lioville's Theorem.
5. State and prove Fundamental theorem of Algebra.
6. Compute n(γ,a) if Z(t)=$e^{4\pi i t}$ where 0$\leq t \leq 2\pi$ and a=0.
7. Compute n(c,0) if Z(t)=$e^{2\pi i t}$ where 0$\leq t \leq 1$.
8. Evaluate $\int_{|Z|=x} \frac{sinZ}{(Z-a)^2}$ dZ.
9. Evaluate $\int_{|Z|=1}^{-1} |Z-1|\,|dZ|$ & $\int_{|Z|=1}$ $e^Z$ $Z^{(-n)}$ dZ.
10. State and prove classical theorem of weirestrass.
11. State and prove Schwartz Lemma.
12. Prove that the limit point of zero's of f(Z)is an isolated singularity.

**UNIT-II**

1. Define chains &cycles, zero chain, simply connected Region.
2. Prove that the differential pdxqdy, where p&q are defined and continuous is a simply connected region Ω if ∫pdx+qdy=0 for every rectangle contained in Ω.
3. Prove that Cauchy theorem for a simply connected Region.
4. Find the residue of f(Z)=$\frac{Z^2}{Z^2+a^2}$ .
5. Find the residue of f(Z)=$\frac{1}{Z^2+5Z+6}$ .
6. Find the residue of f(Z)=tanZ, f(Z)=cotZ f(Z) =$\frac{1}{sinZ}$ .
7. State and prove cauchy Residue theorem
8. State and prove Argument Principle theorem
9. State and prove Rouche's theoremHow many zeros of the equation Z 6 -6Z 3 +3=0 lie in the annuals 1
10. How many zeros of the equation Z 4 -6Z 2 +3Z+1=0 le in the annuals 1 .

11. How many roots does the equation Z 7 -2Z 5 +6Z 3 -Z+1=0 having in the disk
12. How many roots of the equation Z 4 -6Z 2 +3Z+1=0 lie in 1.
13. How many zeros of the equation z6-6z 3 +3=0 lie in the annulus 1.
14. Prove that z4+4(1+i)z+4=0 has one zero in 2 nd audrant.
15. 16. How many roots of the equation z4+8z 3 +3z 2 +8z+3=0 lie in the right

## UNIT III

1. Evaluate $\int_0^{\pi} \frac{d\theta}{a+cos\theta}$ , a>1

2. State & Prove Schwartz's formula.

## UNIT IV

1. Show that any elliptic function with periods $\omega_1$ ,$\omega_2$ can be expressed as

$$c\prod_{k=1}^{n} \frac{p(z)-p(a_k)}{p(z)-p(b_k)}$$

2. Find the taylor series expansion of sinz

## UNIT V

1. State & Prove Jensen's Formula
2. State & Prove Mittag Leffer's Theorem & solve $\frac{\pi^2}{sin^2 \pi^2} = \sum_{-\infty}^{\infty} \frac{1}{z}$

**UNIT-1 SECTION-A (10 MARKS QUESTIONS )**

1. If f(a) and all its derivative vanishes is taylor's theorem then there exist a neighbourhood of a point a throughout which the function f(z) vanishes completely.

2. state and prove maximum modulus principle theorem

UNIT-II

1.Prove that a region Ω is simply connected if and only if h(γ,a) for all cycles γ in Ω and all points 'a' which are not belong in Ω

2. state and prove General statement of cauchy's theorem

3. state and prove open Mapping theorem

UNIT-3

1. Evaluate $\int_0^{\pi} \frac{d\theta}{a+cos\theta}$ , a> 1
2. If u(2) is harmoniz in $|z| < R$ ϕ continuous in the region |z|≤ $R$ then

   u(a)=$\frac{1}{2\Pi}\int_{|z|} \frac{R^2-|a|^2}{|z-a|^2}$ u(2) d$\theta$ , ∀|a|<R in polar coordinates

   u(r$e^{i\Phi}$)=$\frac{1}{2\Pi}\int_0^{2\Pi} \frac{R^2-r^2 u(Re^{i\theta})d\theta}{R^2-2Rrcos(\theta-\theta)+r^2}$ .
3. Suppose that f(Z)is analyfic in the analytic in the annulms $r_1 < |z| < r_2$ and continuous on the closed annulms of M(r) denotes the maximum of $|f(z)|$ for |z|=r show that M(r) $\leq M(r_1)^{\alpha}$ M$(r_2)^{1-\alpha}$ where $\alpha$ =$\frac{\log(\frac{r2}{r1})}{\log(\frac{r2}{r1})}$ .
4. State and prove argument principle theorem and Rouchy's theorem.
5. (i) Evaluate $\int_0^{\infty}(1+x^2)^{-1}\log x \; dx.$

   (ii) prove that if u is harmonic in Ω then f(Z) =$\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$ is analytic in Ω..
6. Prove that $\frac{1}{2\pi}\int_{|Z|=r} u d\theta = \alpha \log r + \beta$ .

**UNIT-IV**

1. State and prove Taylor's series.
2. State and prove Laernt's series.
3. (i) state and prove Wierstrass theorem.

   (ii) state and prove Hurwitz theorem.
4. State and prove poisson integral formula.

**UNIT-V**

1. State and prove Mittag – Lefler's Theorem.

2. State and prove Weierstrass theorem on entire function.
3. Prove that the Euler constant is 0.57722.
4. State and prove Legendre’s duplication formula.
5. State and prove Jensen’s formula.
6. State and prove poisson’s Jensen’s formula.
7. State and prove Hadamerd’s theorem.