D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1.

COMPLEX ANALYSIS II

UNIT I SECTION A 6 MARKS

1. Prove that the Reimann Zeta function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at s=1 with residue 1.

2. Let F be a family of analytic functions with respect to C. If the function in F are uniformly bounded on every compact set then Prove that F is normal.
3. Prove that the family F is totally bounded if and only of to every compact set $E \subset \Omega$ and every ε>0it is possible find $f_1, f_2,..., f_n \in F$ such that every $f \in F$ satisfies $d(f, f_j) <$ε on E for some f_i
4. A family f of analytic Ffunctions is normal with respect to c if and only if the functions in F are uniformly bounded on every compact set.
5. A locally bonded family of analytic functions has locally bounded derivatives.

6. Prove that the function $\xi(s) = \frac{1}{2}s(1-s)\pi^{\frac{-1}{s}}\Gamma(s/2)\zeta(s)$ is an entire function satisfies

$\xi(s) = \xi(1-s)$

UNIT II

1. State and Prove Harnack's principle and Harnack's inequality.

(OR)

2. State and Prove Schwarz – Chirstoffel formula.
3. Explain boundary behavior.
4. Explain use of reflextion principle5. Suppose that the boundary of a simply connected region Ω contains a line segment γ as one sided free boundary arc.then the function f(z) which maps Ωonto the unit disk can be extened to a function which is analytic and one to one on $\Omega \cup \gamma$.
5. Explain the behavior at an angle of conformal mapping of a polygon.

UNIT III

1. Prove that the zeros of the $a_1, a_2, a_3,...a_n$ and $b_1, b_2, b_3,...b_n$ are poles of an elliptic function $a_1, a_2, a_3,...a_n \equiv b_1, b_2, b_3,...b_n \pmod M$
2. Define elliptic functions, unimodular transformations, canonical basis of period module and the fundamental region of unimodular group.

UNIT IV

1. Derive first order differential equation for w=f(z)
2. State and prove Legendre relation.

UNIT V

1. Prove that every section is a homeomorphism.

2. Explain the terms:germs of analytic functions, sheaf of germs of analytic functions.

UNIT I SECTION B 15 MARKS

1. State and Prove Arzela's theorem

2. a)Prove that A locally bounded family of analytic functions has locally bounded derivatives. if a sequence of analytic functions converges in the same sence ,then the limit function is either analytic or identically equal to ∞

UNIT II

1. State and prove Riemann mapping theorem.
2. A family of analytic or meromorphic function f is normal in the classical sence if and only if the expressions $\rho(f) = \dfrac{2|f'(z)|}{1+|f(z)|^2}$

UNIT III

1. State and prove canonical basis thorem.

2. Explain detail about Normality an Compactness.

3. A family Ƒis normal if and only if its closure Ƒ- with respect to the distance function

UNIT IV

1.Define peptic modular fuction and prove that $\lambda(t)$ is a mod function and discuss its behavious.
2. Briefly Expalin Gamma function
3. state and prove jensens formula.
4.State and prove poisson's jennsens formula.

UNIT V

. 1. Prove that A locally bounded family of analytic functions has locally bounded derivatives.

2. if a sequence of analytic functions converges in the same sence , then the limit function is either analytic or identically equal to ∞ A family of analytic or meromorphic function f is normal in the classical sence if and only if the expressions $\rho(f) = \dfrac{2|f'(z)|}{1+|f(z)|^2}$ State and prove monodramy theorem.