D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1.

II- M.Sc. MATHEMATICS

SUB: DIFFERENCE EQUATION **SUB. CODE:15CPMA4D**

UNIT- I DIFFERENCE EQUATION

SECTION-A 6 MARKS

1. Prove that k th order difference of Kth degree polynomial is a constant
2. Prove $P(E)\ (b^n g(n)) = b^n P(bE) g\ (n)$
3. Prove that the following statements
   (i) $\Delta x^k = kx^{k-1}$
   (ii) $\Delta^n x^k = k(k-1)...(k-n+1)x^{k-x}$
   (iii) $\Delta^k x^k = k!$
4. Prove Δ^{-1} is linear , $\Delta^{-1}[ax(n)+by(n) = a\Delta^{-1}x(n)+b\Delta^{-1}y(n)$
5. Solve y(n+3) $-\frac{n}{n+1}y(n+1)$+ny(n+1)-3y(n)= n , y(1)=0, y(2)=-1, y(3)=1, find the value of y(4), y(5), y(6) and y(7)
6. Prove initial value problem have a unique solution
7. Show that the function $3^n, n3^n, n^2 3^n$ for n≥ 1
8. Find w(n) of $3^n, n3^n, n^2 3^n$
9. State and prove Abel's lemma
10. Let x_1(n) and x_2(n) be two solutions of x(n+k)+ $P_1(n)$x(n+k-1)+…+ $P_k(n)x(n)$=0 then the following statements are true
    (i) x(n)= $x_1(n) + x_2(n)$is a solution of above equation
    (ii) $\bar{x}$(n)= ax(n) is a solution of above equation for any constant a
11. Find the solution of x(n+3)-7x(n+2)+16x(n+1)-12x(n)=0 , x(0)=0, x(1)=1,x(2)=2.
12. Find the casoration of the function 5^n, $(3{,}5^{n+2})$, e^n
13. Solve y(n+2)+y(n+1) -12y(n)= n2^n
14. Solve y(n+2)-5y(n+1) +6y(n)=1- n
15. Solve y(n+2)+8y(n+1) +12y(n)= e^n
16. (E-3)(E+2)y(n) = 5. 3^n
17. The condition $1+p_1+p_2 > 0$, $1-p_1+p_2 > 0$, $1-p_2 > 0$ are necessarily and sufficient condition for the equilibrium point of the equation y(n+2) +p_1 y(n+1) + p_2 y(n) =M and y(n+2)+ p_1y(n+1)+ p_2 y(n) =0 is asymptotically stable.

SECTION-B 15 MARKS

1. State and prove Abel's lemma
2. State and prove limiting behavior of solution
3. The condition $1+p_1 + p_2 > 0$, $1-p_1 + p_2 > 0$, $1-p_2 > 0$ are necessarily and sufficient condition for the equilibrium point of the equation y(n+2) +p_1 y(n+1) + p_2 y(n) =M and y(n+2)+ p_1y(n+1)+ p_2 y(n) =0 is asymptotically stable.
4. Find the condition under which the solution of the equation y(n+2)- $\alpha(1+\beta)y(n+1)+\alpha\beta y(n)$ =1, $\alpha,\beta > 0$ converges to the equilibrium points y^* and oscilates about y^*

UNIT- II SECTION-A 6 MARKS

1. Find the solution of the difference system x(n+1)= Ax(n) Where, A= $\begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$
2. State and prove existence and uniqueness solution of x(n+1)= A(n)x(n)
3. State and prove Abel's formula
4. There are k linearly independent solutions of system x(n+1)= A(n)x(n) for n≥ n_0 where A(n)=$a_{ij}(n)$ is a k×k non- singular matrix functions
5. State and prove variation of constant formula
6. Let B be a k× k non singular matrix and let m be any positive integer. Then there exists some k × k matrix C such that C^m = B.
7. For every fundamental matrix $\emptyset(n)$ of a periodic system of equations there exist a non- singular matrix p(n) of periodic N , such that $\emptyset(n)$ = P(n) B^n

SECTION-B 15 MARKS

8. Find the solution of the difference system x(n+1)= Ax(n) Where, A= $\begin{pmatrix} 4 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & 1 & 6 \end{pmatrix}$

9. Solve the system y(n+1) = A(n)y(n)+g(n). Where A(n) = $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$, g(n) = $\begin{bmatrix} n \\ 1 \end{bmatrix}$, y(0) = $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

10. Find the general solution of x(n+1) = Ax(n) where A= $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, x(0)= $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

11. Find the general solution of x(n+1) = Ax(n) where A= $\begin{pmatrix} 4 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & 1 & 6 \end{pmatrix}$

UNIT –III Z- TRANSFORM 6 MARKS

1. Find the z- transform of $\{a^n\}$
2. Find the z- transform of z$(n^2 a^n)$
3. Prove that $z[n^k$x(n)] = $\left(-z\frac{d}{dz}\right)^k \bar{x}(z)$
4. Obtain inverse z- transform $\bar{x}(z) = \frac{z(z+1)}{(z-1)^z}$
5. Solve the difference equation x(n+2)+3x(n+1)+2x(n)= 0 , x(0)=1, x(1)= -4.
6. Solve the difference equation x(n+4)+9x(n+3)+30x(n+2)+44x(n+1)+24x(n)= 0 , x(0)=0, x(1)= 0, x(2)=1, x(3)=-10
7. Solve $\frac{z(z+1)}{(z+2)^2(z\_1)}$
8. Solve the difference equation x(n+3)-x(n+2)+2x(n)= 0 , x(0)=1, x(1)= 1, x(2)=1.

9. Solve $\bar{x}(z) = \dfrac{z(z-1)}{(z-2)^2(z+3)}$

10.Solve $\bar{x}(z) = \dfrac{z(z-1)}{(z+2)^3}$

SECTION-B 15 MARKS

1.If $x_n \in l_1$ then

(i) $\bar{x}(z)$ is an analytic function for $|z| \geq 1$

(ii) $|\bar{x}(z)| \leq \|x\|$, for $|z| \geq 1$

2. Solve the difference equation
x(n+4)+9x(n+3)+30x(n+2)+44x(n+1)+24x(n)= 0 x(0)=0, x(1)= 0, x(2)=1, x(3)=-10

3. The zero solution of x(n+1)= Ax(n) +$\sum_{j=0}^{n} B(n-j)\, x(j)$ uniform ly stable if and only if

(a). Z- A-$\bar{B}(z) \neq 0$ for all |z|>1

(b). if z, is a zero of g(z) with $|z_r| = 1$. Then the residue of g(z) at z_r is bounded as n→ ∞.

4. The zero solution of x(n+1)= Ax(n) +$\sum_{j=1}^{n} B(n-j)\, x(j)$ uniformly asympotically stable is either one of the condition is hold

(i) $\sum_{j=1}^{k}(|a_{ij}| + \beta_{ij}) < 1$ for each i, $1 \leq i \leq k$

(ii) $\sum_{i=1}^{k}(|a_{ij}| + \beta_{ij}) < 1$ for each j, $1 \leq j \leq k$, where $\beta_{ij} = \sum_{n=0}^{\infty}|b_{ij}(n)|$,1 $\leq i, j \leq k$

UNIT – IV

ASYMPTOTIC BEHAVIOUR OF DIFFERENCE EQUATION

SECTION-A 6 – MARKS

1. Show that $t^2 \log t$ $+t^3$ = O(t^3) as n $\to \infty$
2. Suppose that $\lim_{n\to\infty} \frac{x(n+1)}{x(n)} = \lambda,\ \lambda \neq 0$ then x(n) = $\pm\lambda^n e^{nv(n)}$ for sequence v(n)
3. State and prove Benzaid- lutz therom
4. suppose that the matrix A has k linearly independent eigen vector ε_1 ε_2 .... ε_k and k corresponding eigen values λ_1 λ_2 ... λ_k. If $\sum_{n=n_0}^{\infty} \frac{1}{|\lambda_i(n)|} \|B(n)\| < \infty$, hold for B(n) then the system y(n+1) = (A+B(n)) y(n) has solutions $y_i(n)$.

5. Find the asymptotic estimate of a fundamental set of solution y(n+1) = [A+B(n)] y(n)

$$\text{Where A} = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \text{ and B(n) } = \begin{bmatrix} \frac{1}{n^2}+1 & 0 & (0.5)^n \\ 0 & (0.2)^n & 0 \\ e^{-n} & 0 & \frac{\log n}{n^2} \end{bmatrix}$$

6. Suppose that the following assumption old
   (i). The system x(n+1+ = D(n)x(n) possess an ordinary Dichotony
   (ii). $\underset{n\to\infty}{lt}\ y(n) = 0$
   (iii). s $\sum_{N=n_0}^{\infty} \|B(n)\| < \infty$ then for each bounded solution x(n) of the given system there correspondes the bounded solution y(n) of y(n+1) = (B(n) +D(n))y(n)
   Such that y(n)= x(n)+O(1)

SECTION-B 15 MARKS

1. State and prove Poincare theorem
2. State and prove variation of constant formula
3. Find the asymptotic estimate of a fundamental set of solution y(n+1) = [A+B(n)] y(n)

Where A = $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ and B(n) = $\begin{bmatrix} \frac{1}{n^2}+1 & 0 & (0.5)^n \\ 0 & (0.2)^n & 0 \\ e^{-n} & 0 & \frac{\log n}{n^2} \end{bmatrix}$

4. Let x(n) be any non- zero solution of x(n+k) +P_1x(n+k-1)+ P_2x(n+k-2)+…. P_k= 0 then

$\underset{n\to\infty}{lt} \frac{x(n+1)}{x(n)} = \lambda_m$ for some characteristic roots provided that distinct characteristic root have distinct module, moreover if there are two or more distinct root with same modulus then $\underset{n\to\infty}{lt} \frac{x(n+1)}{x(n)} = \lambda_m$ may not exist in general but for particular solution λ_r, λ_j n can always be found, for the result $\underset{n\to\infty}{lt} \frac{x(n+1)}{x(n)} = \lambda_m$ exist and this equal to characteristic λ_n

UNIT- V OSCILLATING SEQUENCE

SECTION-A 6MARKS

1. Suppose that p(n) ≥ 0 and $\lim_{n\to\infty} \sup(p(n)) < \frac{k^k}{(k+1)^{k+1}}$ then x(n+1)-x(n)+p(n)x(n-k) = 0 has non- oscillatory solution.
2. Suppose that b(n) > 0 for n∈ z^+ then every solution x(n) of p(n)x(n+1) +p(n-1)x(n-1) =b(n)x(n) is non –oscillatory iff every solution z(n) if c(n)z(n) +$\frac{1}{z(n-1)}$ =1 for n N for some N > 0
3. State and prove Gyori and Lada's theroem
4. If there exists a subsequence b(n_k) ≤ 0 with $n_k \to \infty$ as k → ∞ then every solution of p(n)x(n+1)+p(n-1)x(n-1)= b(n)x(n) oscillates
5. If c(n) ≥ a(n) > 0 for all n>0 and z(n)>0 is a solution of c(n)z(n)+$\frac{1}{z(n-1)}$ = 1 then the equation a(n)y(n)+ $\frac{1}{y(n-1)}$ = 1 has a solution y(n) ≥ z(n) > 1, for all n∈ z^+
6. If b(n)b(n+1) ≥ 4p^2(n) for $n \geq$ N then every solution of p(n)x(n+1)+p(n-1)x(n-1) =b(n)x(n) is non-oscillatory

SECTION-B 15 MARKS

1. Suppose that $\underset{n\to\infty}{lt} \inf p(n) = p > \frac{k^k}{(k+1)^{k+1}}$ then the following statement hold

   (i) X(n+1)-x(n)+p(n)x(n-k)≤ 0 has no eventually positive solution

   (ii) X(n+1)-x(n)+p(n)x(n-k)≥ 0 has no eventually negative solution

2. State and prove Strum Seperation theorem

3. If b(n)b(n+1)< (4-ξ)$p^2(n)$ for some $\xi > 0$, for all n≥N then every solution of p(n)x(n+1)+p(n-1)x(n-1)=b(n)x(n) is oscillatory.