D.K.M COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

DEPARTMENT OF MATHEMATICS

MAJOR: DISCRETE MATHEMATICS – 15CPMA3B

II M.SC., MATHEMATICS

UNIT I SECTION-A 6 MARKS

1. Define the following Term (i) Lattice (ii) Boolean Algebra (iii) Lattice Homomorphism (iv) Relatively Complemented (v) Chain (vi) Sub lattice (vii) Product lattice (viii) Characteristic function (ix) Duality Principle (x) Filter (xi) Poset
2. Prove that a Lattice L is distributive, if $\forall x, y, z \in L$ Cancellation law holds good $\left.\begin{array}{l} x \wedge y = x \wedge z \\ x \vee y = x \vee z \end{array}\right\} \Rightarrow y = z$
3. Let $f : B_1 \to B_2$ be a homomorphism then prove that (i) $f(0) = 0$ (ii) $f(1) = 1$ (iii) $x \leq y \Rightarrow f(x) \leq f(y)$
4. State and prove De Morgan's law.
5. Let L be a lattice then the following implication hold (i) L is a Boolean algebra $\Rightarrow$ L is relatively complemented.
6. Define distributive lattice with example and prove that every chain is a distributive lattice.
7. Prove that the elements of an arbitrary lattice satisfy the following distributive inequalities.
   $x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)$
   $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$
8. Show that a modular lattice is distributive is distributive if and only if none of its sublattices is isomorphic to the diamond lattice V_3^5
9. State and prove the representation theorem for finite Boolean algebra.
10. Define join-homomorphism, meet-homomorphism and order-homomorphism of lattices. State

the relationship among them.

11. Define the partial order set and represent the poset $(P(\{1, 2, 3\}), \subseteq)$ by Hasse diagram.
12. Let B be a Boolean algebra and let I be a non-empty subset of B then prove that the following conditions are equivalent. (i) I $\leq$ B (ii) For all i, j $\in$ I and b $\in$ B: i+j $\in$ I and b $\leq$ i $\Rightarrow$ b $\in$ I
    (iii) I is the kernel of a Boolean homomorphism from B into another Boolean algebra
13. State and prove modular inequality
14. Show that in a Boolean algebra, the complement of every element is unique
15. Define distributive lattice and complement lattice. Give an example of a lattice which is not a Boolean algebra.

SECTION-B 15 MARKS

1. Prove that the cardinality of a finite Boolean algebra B is always of the form 2^n and B has precisely n atoms. Also prove that any 2 Boolean algebras with the same finite cardinality are isomorphic
2. A Lattice L is modular $\Leftrightarrow$ None of its sublattice is isomorphic to the pentagon lattice V_4^5
3. (i) State and prove Representation theorem (ii) Define the Boolean Polynomial function (iii) Define the system of normal forms
4. Prove that L X M is also a distributive lattice if L, M are distributive
5. (i)Find the disjunctive normal form of $(x_1 \bullet (x_2 + x_3)' + \left((x_1 \bullet x_2) + x_3{}' \right)' \Big) x_1$

   (ii) Apply Quine-Mc clusky method to find the minimal form of d where d is given by $d =$ $wxyz' + wxy'z' + wx'yz + wx'yz' + w'x'yz + w'x'yz' + w'x'y'z$
6. Determine the minimal form of P where
   P =
   $vwxyz + vwxyz' + vwxy'z' + vwx'yz + vwx'y'z + vw'x'y'z + vw'x'y'z' + v'wxyz + v'wxyz' +$
   $v'wxy'z' + v'wx'yz' + v'wx'y'z + v'w'xyz' + v'w'xy'z' + v'w'x'yz' + v'w'x'y'z'$

UNIT II SECTION-A 6 MARKS

1. (i)Determine the symbolic representation of the switching circuit

   $P = (x_1 + x_2 + x_3)(x_1' + x_2)[(x_2x_3 + x_1'x_2) + x_3]$

   (ii)Determine the contact diagram of the switching circuit

   $P = x_1\left[x_2(x_3(x_4 + x_5^1) + x_6) + x_7(x_3 + x_6)x_8^1\right]$
2. Simplify using Karnaugh map method

   $P(x_1, x_2, x_3) = (x_1 + x_2)(x_2 + x_3) + x_1x_2x_3$
3. A Hall light is controlled by 2 switches one upstairs and one downstairs. Design a circuit so that the light can be switched on or off from the upstairs or the downstairs in symbolic represention and contact diagram.
4. In a large room there are electrical switches next to the three doors to operate the central lighting. The three switches operate alternatively ie, each switch on or switch off the lights. Each switch has two positions: either on or off. We denote the switches by x_1, x_2, x_3 and the two possible states of the switches x_i by $a_i \in \{0,1\}$. The light situation in the room is given by the value $\overline{P}(a_1, a_2, a_3) = 0$ $(=1)$ if the lights are off (or on) respectively. We choose $\overline{P}(1,1,1) = 1$. If we operate one or all three switches, the lights stay on. Determine the switching circuits, its symbolic representation and contact diagram.
5. Draw a circuit diagram representing the Boolean expression.
   $[(x \wedge y') \vee (x' \wedge y')] \vee [x' \wedge (y \vee z)]$
6. Define the following

(i) Four important gates (ii) Complemented lattice (iii) Karnaugh map method.

7. Describe half adder.
8. Simplify using Karnaugh map method

   $P(x_1, x_2, x_3) = (x_1 + x_2)(x_2 + x_3) + x_1x_2x_3$
9. Describe the following (i) T – Algebra (ii) Measure on σ algebra (iii) Orthocomplemented lattice (iv) Central element
10. Explain the main aspect of the algebra of switching circuits. Give the diagrams for the switching circuits P = $x_1(x_2(x_3 + x_4) + x_3(x_5 + x_6)$

SECTION-B 15 MARKS

1. Explain (i) Full adder (ii) Half adder of switching circuits
2. (i) What are the methods of simplifying a switching circuit? Explain the same for $P = (x_1' + x_2 + x_3 + x_4)(x_1' + x_2 + x_3 + x_4')(x_1' + x_2' + x_3 + x_4')$

   (ii) Determine the symbolic representation of the circuit given by

$P = (x_1 + x_2 + x_3)(x_1' + x_2)(x_1x_3 + x_1'x_2)(x_2' + x_3)$

3. A motor is supplied by 3 generators. The operation of each generator is monitored by corresponding switching element which closes a circuit as soon as a generator fails. We demand the following conditions from electrical monitoring system (i) A warning lamp lights up if one or more generators fails (ii) An acoustic alarm is initialised if two or all three generators fails. Find the symbolic representation as a mathematical model of this problem

UNIT III SECTION-A 6 MARKS

1. Let F be a finite field of characteristic p. Then prove that F contains p^n elements, where $n = \lfloor F : Z_p \rfloor$.
2. Determine the elements of F_{2^3} where $F_2 = \{0,1\}$.
3. If the field F has p^m elements then F is the splitting field of the polynomial $x^{p^m} - x$.
4. State and prove Mobius inversion formula (Additive form)
5. Determine all the elements of F_2^3

6. Let p be a prime number, let m, n be the natural numbers (i) If F_{p^m} is a subfield of F_{p^n} then m / n (ii) If m / n then $F_{p^m} \cong F_{p^n}$ there is exactly one subfield of F_{p^n} with p^n elements

7. Let F be a finite field with p^n elements Prove the following:

(i) The multiplicative group of the non-zero elements of F is cyclic and of order $p^n - 1$.

(ii) All elements a of F satisfies $a^{p^n} - a = 0$

8. Show that in a group each element has precisely one inverse

9. Let L be a finite extension of K and let K be a finite extension of F then prove that [L : K]

[K : F] = [L : F] for a field F of characteristic p, prove that F contains p^n elements where $n = \lfloor F : Z_p \rfloor$

10. (i) State and prove Kronecker's theorem (ii) Define the cyclotomic polynomial

11. Define the following (i) Symmetric group (ii) Subgroup (iii) Maximal ideal (iv) Subfield

12. (i) Define a group (ii) State the statements of lagrange's theorem, Little Fermat's theorem and Unique factorisation theorem

13. Find the sum and the product of $f(x) = 2x^3 + 4x^2 + 3x + 2$ & $g(x) = 3x^4 + 2x + 4$ given that f(x), g(x) in $Z_5[x]$

14. Prove that a polynomial f over F has no multiple zeros in its splitting field if and only if

gcd(f, f^1) = 1

SECTION-B 15 MARKS

1. Prove the following: (i) For every prime p& every positive integer n, there exist a field having p^n elements. (ii) Any field of order p^n is the splitting field of $x^{p^n} - x \in Z_p[n]$

   (iii) Any two fields of order p^n are isomorphic.

2. (i) If F is a finite field with p^n elements, then show that the multiplicative group of the non-zero elements of F is cyclic of order $p^n - 1$ (ii) (ii) Factorise $x^{15} - 1$ over F_2

3. Write the algorithm to compute the GCD of the polynomials f and g. using this find the GCD of f = $2x^5 + 2x^4 + x^2 + x$, g = $x^3 + x^2 + x + 1$ in $Z_3[x]$

UNIT IV SECTION-B 6 MARKS

1. Let f be an irreducible polynomial of degree k over F_q, $f / x^{q^n} - x \Leftrightarrow k/n$
2. Explain Berlekamp's algorithm.
3. State and prove Chinese remainder theorem.
4. Determine the primitive polynomial over F_3 of degree 4.
5. Compute I (2, 4)
6. Define the exponent polynomial and primitive polynomial
7. Define decomposable polynomial &Cyclotomiccoset
8. Let f $\in F_q[x]$ de a polynomial of degree m ≥ 1 with f(0) $\neq 0$ then there exist a positive integer e $\leq q^{m-1}$ such that $f / x^e - x$ prove it

SECTION-B 15 MARKS

1. Show that the polynomial $f \in F_q[x]$ of degree k is primitive over F_q if and only if $f$ismonic, $f(0) \neq 0$ and the order of$f$is equal to $q^m - 1$.
2. Factorise the polynomial $x^8 + x^6 + x^4 + x^3 + 1$ over F_2
3. Determine the complete factorisation of g = $x^8 + x^6 + 2x^4 + 2x^3 + 3x^2 + 2x$ over F_5 by Berlekamp's algorithm
4. State and prove Little Fermat's theorem and Wilson's theorem

UNIT V SECTION-A 6 MARKS

1. Define linear code and generator matrix. Give an example for each of them.
2. For the following parity check matrix: $H = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$

   (i) How many code words does it contain?

   (ii) Give a systematic generator matrix of the code.
3. State and prove an elegant characterisation of cyclic codes.
4. Explain encoding and decoding with an example
5. Define the following (i) Hamming distance (ii) Sphere of radius r (iii) Cyclic code (iv) Syndrome (v) Linear code
6. (i) Define a Hamming distance (ii) State the Shannon's theorem (iii) Let G be a generator matrix of a linear code C then prove that the rows of G forms a basis of C
7. Define (i) Hamming weight (ii) Dual code

SECTION-B 15 MARKS

1. A linear code $C \subseteq F_2^5$ is defined by the generator matrix $G = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{pmatrix}$.

   Determine the rank of *G*, the minimum distance of *C*, a parity-check matrix for *C* and all the code words.

2. Let $H = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$ be the parity check matrix for a Hamming (7, 4) code.

   (i) Encode the following messages 1000, 1100, 1011, 1110, 1001, 1111

   (ii) Decode the following the following received words 1100001, 1110111, 0010001, 0011100

   Using the following table consisting of the syndromes and coset leaders for the code

syndrome	000	001	010	100	011	101	110	111
Coset leader	0000000	0000001	0000010	0000100	0010000	0100000	1000000	0001000

3. (i) Write the decoding algorithm (ii) Define the Hamming code (iii) Define the canonical generator matrix with an example (iv) State the Plotkin bound
4. (i) State Shannon's theorem (ii) Define maximal cyclic codes (iii) Define a cyclic codes (iv) Determine all code words of a code with generator polynomial $g = 1 + x + x^2$ over F_2 if the length of k of the message 4 which of the following received words have detectable errors 1000111, 0110011, 0100011
5. Let C be non-zero ideal in V_n. Then prove that there exists a unique g in V_n with the following properties (i) g divides $x^n - 1$in $F_q[x]$ (ii) c = (g) (iii) g is monic
6. Let h be a check polynomial of a cyclic code $c \subseteq V_n$ with generator polynomial g and $v \in V_n$

   Then prove that $v \in C$ if and only if $v*h = 0$