D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1.

III B.Sc Mathematics

Paper VIII- Dynamics

Question Bank

Unit-I Section – A

1. Define Displacement, Velocity and acceleration of a moving particle.
2. Define Resultant Velocity.
3. Define Relative Velocity.
4. Define Relative angular Velocity.
5. Define Angular Velocity of a moving particle.
6. Define Rectilinear motion.
7. State Newton's law of motion.
8. Define average velocity.
9. Define Uniform velocity.
10. The angular velocity of a moving particle is constant. Prove that its transverse acceleration is proportional to its radial velocity.

Section-B

1. Derive the equatorial of motion.
   a) $v= u+ft$
   b) $s=ut+1/2ft^2$
   c) $v^2-u^2=2fs$
2. Find the velocity and acceleration components of a moving particle along two mutually perpendicular directions.
3. Find the components of the velocity and acceleration of a moving particle along the tangential and normal directions.
4. A particle moves along a straight line such that its radial velocity is k times its transverse velocity. Prove that the path of the particle is an equi-angular spiral.
5. Two particles A and B describe concentric circles of radii 2r and r respectively. The particle move with velocities v and 2v respectively about the centre O. If the relative angular velocity of A with respect to B vanishes, prove that $\cot\alpha=2$, where $\llcorner OAB=\alpha$ and $\llcorner OBA=\beta$.
6. A particle moves along a straight line with initial velocity u and constant acceleration .
   so that the distance travelled by the particle during n^{th} second is $\alpha/2(2n-1)$. If a, b, c denote the distance travelled by the particle during p^{th}, q^{th}, and r^{th} seconds, then prove that
   $a(q-r)+b(r-p)+c(p-q)=0$
7. Two ends of the same train moves such that the end A moves with velocity u and the end B moves with velocity v. Prove that the velocity with which the middle of the train moves is $\sqrt{1/2(u^2+v^2)}$.

8. A particles has simultaneously two velocities v_1 and v_2. Their resultant velocity is equal to v_1 in magnitude. When v_1 is doubled, prove that their resultant velocity is perpendicular to v_2.
9. A particle moves along a straight line with uniform acceleration f and initial velocity u such that it covers equal distance s in successive intervals of time t_1 and t_2 respectively. Prove that $f=2s(t_1-t_2)/t_1t_2(t_1+t_2)$.

Section-C

1. Find the components of velocity and acceleration of a moving particle in polar co-ordinates.
2. A particle P moves along a straight line with uniform acceleration of such that v_1, v_2, v_3 denote the average velocities of P in the consecutive time t1, t2, t3 respectively. Prove that $v_1-v_2/v_2-v_3= t_1+t_2/t_2+t_3$
3. Two planets A and B describe Concentric circles of radii a and b respectively. The planets move with angular speeds v1 and v2 about the sun as centre O. The angular speeds vary inversely as the square root of the radii. If the relative angular velocity of A with respect to B vanishes, prove that cos $\angle AOB= \sqrt{ab}/a-\sqrt{ab}+b$
4. A particle moves with uniform acceleration f along a straight line such that it covers equal distances in successive intervals of time t_1, t_2,and t_3 respectively. Then, prove that $1/t_1-1/t_2+1/t_3=3/t_1+t_2+t_3$
5. A train starts from rest and travels with the constant acceleration α then with the constant velocity v and then with the constant retardation β to come to rest. If s is the total distance travelled, then the total time taken to cover s is $T=s/v+v/2(1/\alpha+1/\beta)$
6. A lift starts from rest and ascends with constant acceleration a then with constant velocity and then with constant retardation a to come to rest. If s is the total distance travelled and T is the total time taken, then the time with which the lift moves with constant velocity is $\sqrt{T^2-4s/a}$

Unit II **Section A**

1. Define a Projectile.
2. Define a trajectory.
3. Define H.R. of a Projectile.
4. Define Maximum height of a Projectile.
5. Define Time of flight of a Projectile.
6. Write down the expressions for H.R., Time of flight and maximum height of a Projectile.

7. Given the Horizontal Range R and Velocity u, when R is maximum? Write the maximum value of R.

8. Write the Maximum value of H.R. on an inclined Palne.

9. Find α when the H.R. is $\frac{u^2}{2g}$ and the initial velocity is u.

10. Define velocity of Projection and angle of projection.

Section B

1. Derive the expressions for H.R, time of flight and maximum height of a projectile.
2. A ball is projected so as to clear two parallel walls. The first wall is of height a and it is at a distance b from the point of projection O. The second wall is of height b and it is at a distance a from O. The ball lies on the plane perpendicular to both the walls. Find the H.R and prove that the angle of projection exceeds $\tan^{-1}(3)$.
3. A Particle is projected over a triangle from one end of the horizontal base to grace the vertex and falls on the other end. If B and C are the base angles and α is the angle of projection, Prove that

   (i) $\tan\alpha = \tan B + \tan C$

   (ii) $1 = \tan\alpha + \tan\beta$ if $\alpha = 45^0$ and $\angle B = \angle\alpha$ and $\angle C = \angle\beta$.
4. If R denotes the H.R and H denotes the maximum height of a projectile, Prove that the velocity of Project is $\sqrt{2gh + \frac{gR^2}{8H}}$.
5. Given u, H.R .R, maximum height H and time of flight T , Prove that i) $16gH^2 - 8Hu^2 + gR^2 = 0$

   ii) $g^2T^4 - 4u^2T^2 + 4R^2 = 0$
6. A Particle is projected from a point O from the ground to clear a wall of height h which is at a distance a from O. If R is the H.R, Prove that $\tan\alpha = \frac{Rh}{a(R-a)}$ where α is the angle of projection.
7. A Particle projected from a point O with the velocity u, making an angle α with OX and passes through the points $P(x_1,y_1)$ and $Q(x_2,y_2)$. Prove that $\tan\alpha = \frac{x_2^2y_1 - x_1^2y_2}{x_1x_2(x_2-x_1)}$.
8. A Particle is projected with initial velocity u by making an angle 45^0 with OX to clear the wall of height h and afterwards strikes the ground at a horizontal distance q on the otherside. If ρ is the horizontal distance covered by the particle before it reaches the height h, Prove that h= pq/p+q.

Section C

1. S.T the Path of the projectile is a Parabola.
2. Derive the H.R of a Projectile on an inclined plane. When R is maximum on an inclined plane.
3. Let v_1 and v_2 be the velocities of a Projectile at the ends of a focal chord of its path. Let v be the velocity of the Particle at the vertex of the focal chord. Prove that $\frac{1}{v_1^2}+\frac{1}{v_2^2}=\frac{1}{v^2}$.
4. A Particle is Projected from a point O with initial velocity u by making an angle α with OX. Prove that i) the particle ceases to rebound after a time $\frac{2usin\alpha}{g(1-e)}$ where e is the coefficient of restitution.

   ii) The Horizontal distance covered by the projectile is $\frac{u^2 sin2\alpha}{g(1-e)}$
5. An elastic sphere is projected from a point O with the initial velocity u by making an angle α with OX. After hitting a smooth vertical wall, which is at a horizontal distance d from o, the sphere comes to rest. Prove that d= $\frac{u^2 sin2\alpha\ .e}{g\ \ (1+e)}$.
6. Find the velocity of a projectile in magnitude and direction by the end of time T. Also Prove that the velocity of the projectile is the same distance as it would be acquired by it falling freely from the verticle distance from the directrix to that point.
7. Prove that greatest height of the Parabola is $\frac{u^2}{2g}-\frac{ga^2}{2u^2}$, where u is the initial velocity, a is the horizontal distance covered.
8. A Particle is projected on an inclined plane of inclination α at an elevation β with OX. Prove that i) cot β= 2tan(α-β) of the particle strikes the inclined plane at right angles.

   ii) tan α = 2tanβ if the particle strikes the inclined plane horizontally.

Unit III **Section A**

1. Define Impact of a force.
2. Define Direct impact of two smooth spheres.
3. State Newton's experimental law of impact.
4. State the Principal of conservation of Linear momentum.
5. Two perfectly elastic spheres impinch each other directly what can you say about their velocities after impact.
6. Define SHM.
7. Define period, frequency of SHM.
8. What are the maximum velocity and maximum acceleration of SHM?

9. Define epoch of SHM.
10. Define Vibration and oscillation of SHM.

Section B

1. State the Laws of Impact.
2. Derive the velocity and displacement equations of SHM.
3. In direct impact, find the impact of blow of m on m!.
4. An elastic sphere of mass m impinches obliquely with smooth fixed plane. Find the magnitude and direction of velocity of m after impact.
5. 2 equal balls B and C are in contact on a table. A third equal ball strikes both of them symmetrically. After impact, the third ball comes to rest. Prove that e=2/3.
6. A Particle moves along a straight line with velocity equation $v^2=\alpha-\beta x^2$ (α,β are constants). Show that the motion is SHM. Also find its period and amplitude.
7. A Particle moves with SHM and the distance covered in three consecutive seconds is x_1,x_2,x_3respectively. Prove that T=$\frac{2\pi}{cos^{-1}\left(\frac{x_1+x_2}{2x_2}\right)}$.

Section C

1. Find the loss in K.E due to direct impact of two spheres.
2. Find the loss in K.E due to oblique impact of two spheres.
3. A ball is thrown from a height h on a horizontal plane and bounces up and down. Prove that
   i) The total vertical distance covered is $h\left(\frac{1+e2}{1-e2}\right)$.
   ii) The total time taken to cover the vertical distance is $\left(\frac{1+e}{1-e}\right)\sqrt{\frac{2R}{g}}$.
4. Find the resultant of two SHM of same period moving along
   i) the same straight line ii)Perpendicular lines
5. A heavy particle of mass m is attached to an elastic string of length l and coefficient of elasticity
   $h = \left(\frac{mg}{k}\right)$. The upper end of the string is fixed and the particle is allowed to fall freely from the lower end particle is allowed to fall freely from the lower end. Prove that the greatest speed of the particle is $\sqrt{gl(2+k)}$.

UNIT IV SECTION A

1. Define central force and central orbit
2. Write down the polar equations of the central orbit.
3. Write down the pedal equations of the central orbit.

4. Define Apse and Apsidal distance.
5. State Kepler's laws of planetary motion.

SECTION B

1. P.T the areal velocity of a moving particle is constant.
2. A planet describes an elliptical orbit with sun as one of its focii.Show that the velocity of the planet when it is away from the sun is greatest and is given by $\frac{2\pi ae}{T\sqrt{1-e^2}}$ where 2a is the length of the major axis.
3. Find the periodic time of an ellipse when a particle moves in a central orbit with an attractive central force $\bar{F}$.
4. A Particle moves under an attractive central force $\bar{F}$=μu^3 and it is projected from an apse at a point 'a' from the pole O by making an angle $\pi/4$ with OX at a distance a with the velocity $\sqrt{\mu}/a$. Prove that the path of the particle is an equi-angular spiral r=a$e^{-\theta}$.
5. Find the central force $\bar{F}$ given the central orbit r =a$e^{\theta cot\alpha}$.
6. Find the central force $\bar{F}$ given the central orbit $r^n = a^n \cos n\theta$.
7. Find the central force $\bar{F}$ given the central orbit $r^n = Acosn\theta - Bsinn\theta$.
8. Find the central force $\bar{F}$ given the central orbit $\frac{l}{r}$= 1+ ecosθ.
9. State and prove the differential equation of the central orbit in polar co-ordinates.
10. State and prove the equation of the central orbit in pedal form.

Section C

1. Find the velocity components of each conic $\frac{l}{r}$=1+ ecosθ.
2. A particle described the central orbit with an attractive central force $\bar{F}$ =$5\mu u^3+8\mu c^2h^5$ and it is projected from an apse at a distance c from the pole O. Find the central orbit when the velocity is $2\sqrt{\mu}/c$.
3. A Particle described the central orbit with an attractive central force $\bar{F}$=$\mu[3au^4-2(a^2-b^2)u^5]$ and it is projected from an apse at a distance (a+b) from the pole O. Find the central orbit when the velocity is $\sqrt{\mu}/(a+b)$.
4. Find the central orbit of a moving particle which moves under an attractive central force
$\bar{F}$ such that its acceleration varies as the distance from the fixed point O.
5. A particle describes a central orbit with an attractive central force $\bar{F}$= $\mu(r^5-c^4r)$ and it projected from an apse at a distance c from O with the velocity $\sqrt{2\mu/3}$ c^3. Prove that the path of the central orbit is $x^4+y^4=c^4$.

Unit V **Section A**

1. Define moment of inertia of a moving particle.
2. Define Radius of Gyration.
3. State perpendicular axes theorem.
4. Write down the M.I of a solid sphere of radius 'a' about its tangent line.

5. Write down the M.I of a hollow sphere of radius ‘a’ about its tangent line.
6. Find the M.I of a thin uniform rod of length 2a.
7. Write down the M.I of a circular lamina of radius ‘a’ about its tangent line.
8. Write down the M.I of an elliptic lamina about its major and minor axes.

Section B

1. Show that M.I of a hollow sphere about its diameter is $\frac{2m}{5}\left(\frac{a^5-b^5}{a^3-b^3}\right)$ where M is the mass of the hollow sphere, a is the external radius of the solid sphere and b is the internal radius of the solid sphere.
2. Show that the M.I of the truncated cone about its axis is $\frac{3m}{10}\left(\frac{a^5-b^5}{a^3-b^3}\right)$ where a and b are the radii of the ends of the cone.
3. Prove that the M.I of a triangular lamina ABC of mass M about its base BC is $Mh^2/6$ where h is the altitude from the opposite vertex.

Section C

1. Find the moment of inertia of an ellipse lamina of mass m.
2. Find the moment of inertia of a solid right circular cone of height h
   i)about its axis ii) about its base diameter.