

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1.

LAPLACE AND FOURIER TRANSFORMS

UNIT-I SECTION-A 2 MARKS

1. Define Laplace transforms.
2. Define Exponential Order.
3. Find $L[t]$.
4. Find $L[t^2]$.
5. Find $L[t^n]$.
6. Find $L[e^{-at}]$.
7. Find $L[\sin at]$.
8. Find $L[\cos at]$.
9. Find $L[\sinh t]$.
10. Find $L[\cosh t]$.
11. Find $L[\sin^2 5t]$.
12. Find $L[\cos^2 5t]$.
13. Find $L[\cos(at+b)]$.
14. Find $L[8e^{8t} + \cosh 3t + \sin 5t]$.
15. Find $L[\sin 5t \cos 2t]$.
16. Define Linearity Property.
17. Find $L[\sin t \sin 2t \sin 3t]$.
18. Find $L[6e^{-st} - t^2 + 2t - 8]$.
19. Find $L[\sqrt{t}]$.
20. Find $L[t^{\frac{3}{2}}]$.
21. Find $L[e^{-at} \cos bt]$.
22. Show that the function $f(t) = e^{t^2}$ is not of exponential order.
23. Find $L\left[\frac{\cos at}{t}\right]$.

SECTION-B 5 MARKS

24. Prove that $L[t^n] = \frac{\sqrt{n+1}}{s^{n+1}}$ where 'n' is a positive integer.

25. Find $L[\sin^2 t \cos 3t]$.

26. Prove that If $L[f(t)] = F(s)$, then $L[tf(t)] = -\frac{d}{ds}F(s)$.

27. find $L[t e^{-4t} \sin 3t]$.

28. Find $L[t^2 e^{-t} \cos t]$.

29. Find $L[S \int_0^t k_s dt]$.

30. If $L[f(t)] = F(s)$ and if $\frac{f(t)}{t}$ has a limit as $t \rightarrow 0$ then $L[\frac{f(t)}{t}] = \int_s^\infty F(s) ds$.

31. Prove that $L[f'(t)] = s L[f(t)] - f(0)$.

SECTION-C 10 MARKS

32. State and Prove shifting Theorem.

33. Find $L[t^2 e^t \sin t]$.

34. Find $L[\frac{1-\cos t}{t^2}]$.

35. Find $L[\frac{\sin at}{t}]$ Hence Showed that $\int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}$

UNIT-II

SECTION-A

2 MARKS

1. Define Periodic functions.

2. State Initial Value Theorem.

3. State Final Value Theorem.

4. Define Laplace transform of the periodic functions.

5. Find $L[\frac{\sin ht}{t}]$.

6. Find $L[\frac{1-e^t}{t}]$.

7. Define Inverse Laplace Transforms.

8. Find $L^{-1}[\frac{a}{s^2+a^2}]$.

9. Find $L^{-1}[\frac{s}{s^2+a^2}]$.

10. Find $L^{-1}\left[\frac{1}{(s-a)^2+b^2}\right]$.

11. Find $L^{-1}\left[\left[\frac{s}{(s^2+a^2)^2}\right]\right]$.

12. Find $L^{-1}\left[\frac{s^2-a^2}{(s^2+a^2)^2}\right]$.

13. Find $L^{-1}\left[\frac{1}{s-3}\right]$, $L^{-1}\left[\frac{1}{s^2-25}\right]$, $L^{-1}\left[\frac{2s}{s^2-16}\right]$.

14. Find $L^{-1}\left[\frac{s+2}{(s+2)^2-36}\right]$.

15. Find $L^{-1}\left[\frac{2s^2-4s+5}{s^3}\right]$.

16. Find $L^{-1}\left[\frac{1}{(s+1)^2+1}\right]$.

17. Find $L^{-1}\left[\frac{2s-5}{9s^2-25}\right]$.

18. Find $L^{-1}\left[\frac{3s}{2s+9}\right]$.

SECTION-B 5 MARKS

1. State and prove Initial Value Theorem.

2. State and Prove Final Value Theorem.

3. Verify the initial and final value theorem for $f(t)=1-e^{-at}$.

4. Find $L\left[\frac{1-\cos at}{t}\right]$.

5. Verify the initial and final value theorem for $f(t)=e^{-t} \sin t$.

6. Find the Laplace transform of rectangular wave given by $f(t)=\begin{cases} 1, & 0 < t < b \\ -1, & b < t < 2b \end{cases}$ with $f(t+2b)=f(t)$.

7. Find the Laplace transform of $|sint|$.

8. Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$.

9. Find $L^{-1}\left[\frac{s}{(s+2)^2+4}\right]$.

10. Find $L^{-1}\left[\frac{s^2}{(s-2)^3}\right]$.

11. Find $L^{-1}\left[\frac{1}{s(s+3)}\right]$.

12. Find $L^{-1}\left[\frac{1}{s^2(s+a)}\right]$.

13. Find $L^{-1}\left[\frac{s^2+3}{s(s^2+9)}\right]$.

14. Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$.

15. Find $L^{-1}\left[\frac{s-3}{s^3+4s+13}\right]$.

16. Find $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$ by using Convolution Theorem.

17. Find $L^{-1}\left[\frac{s}{(s^2+2s+s)^2}\right]$ by using Convolution Theorem.

18. Find $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right]$ by using Convolution Theorem.

19. Find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ by using Convolution Theorem.

SECTION-C 10 MARKS

1. State and prove initial & final value theorem.
2. Verify the initial and final value theorem for the function $f(t)=1+e^t(\sin t+\cos t)$.
3. Verify the initial and final value theorem for $f(t)=3 e^{-2t}$.

4. Find the Laplace transform of the Half sine wave rectifier function

$$f(t) = \begin{cases} \sin wt, & 0 < t < \pi/w \\ 0, & \pi/w < t < 2\pi/w \end{cases}$$

5. Find the Laplace transform of $f(t) = \begin{cases} t, & 0 < t < b \\ 2b - t, & b < t < 2b \end{cases}$ with $f(t+2b) = f(t)$.

6. Find $L^{-1}\left[\frac{5s^2-15s-11}{(s+1)(s-2)^3}\right]$.

7. Find $L^{-1}\left[\frac{1-s}{(s+1)(s^2+4s+13)}\right]$.

8. Find $L^{-1}\left[\frac{s+2}{(s^2+4s+13)^2}\right]$.

UNIT-III SECTION-A 2 MARKS

1. Define fourier Integral theorem.
2. Define Integrals Equation of Convolution.
3. Write the Application to solve differential equations.
4. What is the formula for solving simultaneous equations.

SECTION-B 5 MARKS

5. Solve the integral equation $Y(t) = 1 + \int_0^t Y(u) \sin(t-u) du.$

6. Using Laplace Transform solve $(D^2 + 2D - 3)y = \sin t$ given that $Y = \frac{dy}{dt} = 0$, when $t=0$.

SECTION-C 10 MARKS

7. Using Laplace transform, Solve $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = e^{-2t}$ Given that $y=0, \frac{dy}{dt} = 1$, when $t=0$.

8. Using Laplace Transform $\frac{d^2y}{dx^2} + 4y = 5e^{-x}$ Given that $y(0)=2, y'(0)=3$.

9. Using Laplace transform Solve $\frac{d^2y}{dx^2} - y = x^2 + x$ Given that $y(0)=y'(0)=0$.

10. Solve the simultaneous equations $3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1, \frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$, Given $x=0, y=0$ at $t=0$.

11. Solve $\frac{dx}{dt} - 2x + 3y = 0, \frac{dy}{dt} - y + 2x = 0$ with $x(0)=8, y(0)=3$.

UNIT-IV SECTION-A 2 MARKS

1. Define Fourier Integral theorem.

2. Define Fourier Transform.

3. Write Fourier Cosine and Sine transform pair.

4. Define Convolution theorem for Fourier transform.

5. Define Parseval's Identity.

SECTION-B 5 MARKS

1. Find Fourier Cosine integral of the functions e^{-ax} . Hence deduce the value of the integral $\int_0^\infty \frac{\cos \lambda x}{1+\lambda^2} d\lambda$.

2. Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} x, & |x| \leq a \\ 0, & |x| > a \end{cases}$.

3. Show that the Fourier transform of $e^{-x^2/2}$, is $e^{-s^2/2}$.

4. Find Fourier sine transform of e^{-x} $x \geq 0$, Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$.

5. Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$.

6. Find the sine transform of $f(x) = \frac{e^{-ax}}{x}$.

SECTION-C**10 MARKS**

1. Find fourier Cosine transform of $\frac{1}{x^2+a^2}$.

2. Find the fourier Sine transform of $f(x)$ if $f(x) = \frac{1}{x(a^2+x^2)}$.

3. Find the fourier sine and cosine transform of x^{n-1} and hence prove that $\frac{1}{\sqrt{x}}$ is self reciprocal. Under fourier sine and cosine transform.

4. Find the F.T of $f(x)$ is given by $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$. Hence Show That $\int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$.

5. Find the F.S.I formula for $\frac{\pi}{2} e^{-x}$.

6. Find the F.T of fuction $f(x)$ define by $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$, Hence Prove that $\int_0^\infty \frac{\sin s - s \cos s}{s^3} \text{Cos}(s/2) ds = \frac{3\pi}{16}$.

UNIT-V SECTION-A 2 MARKS

1. Define Linear Property.

2. Prove that $F[e^{iax} f(x)] = F(s + a)$

3. Prove that $[\int_a^x f(x)dx] = \frac{F(s)}{(-is)}$.

4. Prove that $F[\overline{f(x)}] = \overline{F(-s)}$.

5. Prove that $F_c[f'(x)] = -\sqrt{\frac{2}{\pi}} f(0) + S F_s(s)$.

6. Prove that $\int_0^\infty F_s(f(x)) G_s(g(x)) ds$.

7. Prove that $F_c[x.f(x)] = \frac{d}{ds} F_s(f(x))$.

8. Prove that $F_s(s) = F_s(f(x)) = \int_0^\infty f(x) \sin sx dx$.

SECTION-B 5 MARKS

9. Prove that for any non-zero real a , $F[f(ax)] = \frac{1}{|a|} F(s/a)$.

10. State and Prove Shifting property.

11. State and Prove Modulation theorem.

12. Prove that $F[x^n f(x)] = (-i)^n \frac{d^n F(s)}{ds^n}$.

13. Find fourier Cosine transform of $e^{-a^2 x^2}$ and hence Find $F_s[xe^{-a^2 x^2}]$.

14. Use the Inverse formula to obtain $f(x)$ in $F_s(s) = \frac{s}{1+s^2}$.

15. If $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| \geq a \end{cases}$ and $F(s) = \frac{2 \sin as}{s}$, $s \neq 0$ Prove that $\int_0^\infty \frac{\sin 2x}{x^2} dx = \pi a/2$.

SECTION-C **10 MARKS**

1. Find the fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| \geq a \end{cases}$ hence deduce that

i) $\int_0^\infty \frac{\sin t}{t} dt = \pi/2$ ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \pi/2$.

2. Use Parseval's identity for fourier Cosine and Sine transforms of e^{-ax} evaluate

i) $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$ ii) $\int_0^\infty \frac{x^2}{(a^2+x^2)^2} dx$.

3. Verify Convolution theorem for $f(x)=g(x)=e^{-x^2}$.