D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE.

I B.Sc.- Mathematics
Allied-Mathematical Statistics – II

Unit – I - Distributions

SECTION -A

1. Define binomial distribution.
2. Show that the mean of the binomial distribution is np.
3. Comment on the following:
   X follows a binomial distribution with mean 3 and variance 9.
4. Determine the binomial distribution for which mean is 4 and variance 3.
5. For a binomial distribution mean 6 and standard deviation is $\sqrt{2}$. Find the parameters.
6. A die is thrown 3 times. If getting a 6 is considered a success, find the probability of atleast two successes.
7. Ten coins are thrown simultaneously. Find the probability of getting atleast seven heads.
8. Find p for a binomial variate X, if n=6, and 9P(X=4) = P(X=2).
9. Define Poisson distribution.
10. If P(X = 1) = P(X = 2) for a Poisson variable X, with parameter λ, find E(X).
11. State Additive Property of Poisson random variables.
12. Find the probability that atmost 5 defective fuses will be found in a box of 200 fuses if experience shows that 2 percent of such fuses are defective.
13. Find the characteristic function of Poisson distribution.
14. Define Normal distribution.
15. Write any two properties of Normal distribution
16. In a binomial distribution 15 independent trials if P(X=0)=P(X=1) Find the value of P and q?
17. Comment on the following the mean of binomial distribution is 3 and variance is 4?
18. Find the area under the standard normal curves and area of a which line to the right of z=2.7 and also find left side of z=2.7?

SECTION - B

19. Find MGF of a binomial distribution.
20. Six dice are thrown 729 times. How many times do you expect atleast 3 dice to show 5 or 6?
21. In a large consignment of electric bulbs 10% are defective. A random sample of 20 is taken for inspection. Find the probability that (i) All are good bulbs (II) Atmost there are 3 defective bulbs (iii) Exactly there are 3 defective bulbs.
22. Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) atleast 1 boy (iii) atmost 2 girls and (iv) children of both genders. Assume equal probabilities for boys and girls.
23. Prove that poisson distribution is a limiting case of Binomial distribution. Find its mean and variance.
24. Prove that sum of two independent poisson variates is a poisson variate and difference of two poisson variates is not a poisson variate.
25. Prove that the conditional distribution of X_1 given X_1+X_2 is binomial. Show that the addition of two independent poisson variates is a poisson variate.
26. Derive Recurrence Formula of Binomial distribution for moments.
27. If the independent random variables X and Y are binomially distributed with parameters (3, 1/3) and (5, 1/3) respectively. Find $P(X+Y \geq 1)$
28. Derive mean and variance of Poisson distribution.
29. Derive the Poisson distribution as a limiting from of the Binomial distribution.
30. State and prove additive Property of Poisson random variables.
31. Write the properties of Normal distribution.
32. Obtain the central moments of all odd and even orders for the Normal distribution $N(\mu,\sigma^2)$.
33. Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percent of student scored
    i. More than 60 marks?
    ii. Less than 56 marks?
    iii. Between 45 and 65 marks?

34. Derive MGF of Normal distribution.

35. A multiple choice test consists of 8 questions with 3 answers to each question (of which only one is correct). A student answers each question by rolling a balanced die and chequing the first answer if he gets one or two, the second answer if he gets three or four and the third answer five or six. To get a distinction, the student must sequered atleast 75% correct answers. If there is no negative marking. What is the probability that the student sequres a distinction?

36. A manufacturer of cotter pins knows that 5 percentage of product is defective, if he sales cotter pins in Boxes of 100 and guarantees that not more than 10 pins will be defective. What is the approximate probability that a box will fail to meet them guaranteed quality?

SECTION – C

37. Find MGF of a binomial distribution and hence find mean and variance.

38. Derive Moments of Binomial distribution.

39. In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution?

40. Prove that the limiting case of binomial distribution is Poisson.

41. Derive MGF of Binomial distribution and hence find μ_1, μ_2 *and* μ_3.

42. Find the MGF of a Normal distribution. Hence find its mean and variance.

43. If 10% of the screws produced by an automatic machine are defective, find the probability that of 20 screws selected at random, there are

    i. Exactly two defectives
    ii. At most three defectives
    iii. At least two defectives
    iv. Between one and three defectives (inclusive).

    Find also the mean and variance.

44. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find
 the mean and variance of the distribution.

Unit – II – Sampling Distributions

SECTION -A

1. Define sampling distribution.
2. Define Students t-distribution.
3. What are the assumptions made for student's t-test?
4. Write the applications of t-test.
5. Define Chi-Square distribution.
6. State any two uses of Chi-Square distribution.
7. What is 2x2 contingency table. Give an Example.

SECTION - B

8. What are the parameters and statistics in Sampling?
9. State and prove the additive property of chi-square variates.
10. Find the relation between the moments of the chi-square distribution.
11. Write the properties of 't' distribution.

SECTION – C

12. Derive Chi-square distribution with n – degrees of freedom.
13. Find the Mean deviation about mean for the Chi-square distribution
14. Derive Student's t distribution with n – degrees of freedom.
15. Find the MGF of the chi-square distribution.
16. Show that the limiting case of the chi-square distribution is the normal distribution.
17. If X and Y are independent chi-squares variates with n1,n2 degrees of freedom respectively. Find the distribution followed by U=X$\frac{X}{X+Y}$, V=X+Y.
18. Show that the limiting form of t-distribution is normal distribution.
19. If χ^2 is a chi square variable with n degrees of freedom prove that for large n,

    $\sqrt{2\chi^2}$ is N($\sqrt{2n}$,1)
20. Find the distribution of the quotient of two independent χ^2 variables is a beta distribution second kind.

Unit – III – Testing of Large and Small Samples

SECTION -A

1. Define population and sample.
2. Define parameter and statistic.
3. Define standard error.
4. What is the utility of standard error.
5. Define statistical hypothesis.
6. What are the errors in sampling?
7. Define Null hypothesis.
8. Define critical region.
9. Define critical value.
10. Write the formula for t-test for difference of mean.
11. What is 2x2 contingency table.Give an Example.
12. What are Type I and Type II error.
13. Write down the formula to test the difference between means of small samples when
population standard deviation is not known.
14. Explain method of moments.

SECTION - B

15. The means of two samples of sizes 600 and 800 are 66.5 and 68.3 respectively. Can they be regarded as drawn from a normal population with standard deviation 2.5.
16. The theory predicts that the population of beans in four groups should be 9:3:3:1 . In an examination with 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory?
17. Before an increase in excise duty on tea, 400 people out of 500 persons were found to
be tea drinkers. After an increase in duty, 400 people were tea drinkers in a sample of 600 people. State whether there is a significant decrease in the consumption of tea.
18. A random sample of 200 villages form a district gives the mean population per village
at 485 with S.D. of 50. Another random sample of the same size from the same district gives the mean population per village is 510 with S.D 40. Is the

difference

between the means significant.

19. The machine produces 16 imperfect articles in an sample of 500. After machine is overhauled, it produced 3 imperfect articles in a batch of 100. Has the machine been improved?

20. The following table gives the classification according to gender and the nature of work. Test whether nature of work is independent of the gender.

Gender / Nature of work	Skilled	Unskilled
Male	40	20
Female	10	30

21. Two random samples are drawn from a normal population having the same variance.

Sample 1	20	25	10	11	21	15	6	
Sample 2	25	26	34	30	39	25	33	54

Test whether the two populations have the same mean.

22. Two random samples are drawn from a normal population having the same variance.

Sample 1	20	25	10	11	21	15	6	
Sample 2	25	26	34	30	39	25	33	54

Test whether the two populations have the same mean.

23. Out of 40 patients who were given a particular injection 8 survived. Test this hypothesis that the Survival rate in the population is 85%.

24. The means of two samples of sizes 100 and 200 are 67.5 and 68.3 respectively. Can they be regarded as drawn from the same sample population with S.D. 2.5.

25. A sample of 150 items had S.D of 2.65. Just at 5% level of significant whether there is any significance difference between this value and the population S.D of 3.55 (the population is a normal population).

26. Out of 20 patients who were given a particular injection 18 survive. Test the hypothesis that a survival rate in a population is 85%.

27. In a city A 20% of a random sample of 900 school boys had a slight physical defect. In another city B 18.5% of a random sample of 1600 school boys had the same defect. Test whether the difference between the two proportions is significant at 5% level.
28. A sample of 120 nails is drawn from a population and their lengths are given below. 3.57, 3.58, 3.59, 4, 4.12, 3.89, 3.96, 4.15, 4.11, 3.88, 4.10, 4.02. Assuming normality test whether the population mean can be 4.
29. A random sample of 1000 members had a mean height of 170c.m and standard deviation of 2.6 can the sample be regarded as drawn from a normal with mean height 165c.m
30. The mean of 2 samples of sizes 600 and 800 are 66.5 and 68.3 respectively can they be regarded as drawn from a normal population with standard deviation 2.5.
31. A random sample of 200 tons of coconut oil gave an average weight of 4.95 kgms with S.D 0.21kg. Do we accept the hypothesis 5 kgms at 1% level.
32. A sample of 10 house owners is drawn and the following values are obtained. Mean Rs.6000 and S.D Rs.650.Test the hypothesis that the average income of house owners of the town is Rs5500.

SECTION – C

33. Test whether the machine is working properly.
    4.7, 4.9, 5, 5.1,5.4, 5.2, 4.6, 5.1, 4.6, 4.7.
34. Prove that in a 2 x 2 contingency table with cell frequencies

a	b
c	d

the value of χ^2 on the assumption of independence is given by

$$\chi^2 = \left[\frac{(a+b+c+d)(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}\right]$$

35. The following table gives the number of aircraft accident that occur during the various days of the week.

Days	Sun	Mon	Tue	Wed	Thurs	Fri	Sat	Total
No. of accident	14	16	8	12	11	9	14	84

Find whether the accidents are uniformly distributed over the week.

Unit – IV - Estimations

SECTION -A

1. Define Estimator.
2. Define Point estimation.
3. What is meant by unbiased estimator? Also give an example.
4. Define Sufficient Estimator with an example.
5. Define efficient estimator. Give an example.
6. Define consistent estimator.
7. Explain method of moments.
8. State methods of Estimation.
9. Obtain an estimator for the parameter p by the method of moments in case of binomial distribution.
10. Obtain an estimator for the parameter λ by the method of moments in case of Poisson distribution.

SECTION - B

11. Let $x_1, x_2, \ldots\ x_n$ be a simple random sample from a normal population with μ and variance 1. Show that the sample mean is an consistent estimator of the population mean μ.
12. If $E(t_n) = \theta$, Var $t_n \to 0$ as $n \to \infty$ then t_n is a consistent estimator of θ.
13. If $x_1, x_2, \ldots\ x_n$ is a simple random sample from a normal population with μ and variance σ^2. Show that the sample variance is a consistent estimator of the population variance.
14. Let $x_1, x_2, \ldots\ x_n$ be a random sample from a normal population with μ and variance σ^2. Show that the sample variance is an unbiased estimator of the population variance. Also show that $S^2 = \frac{1}{n-1}\sum(x_i - \overline{X})^2$ is an unbiased estimator of σ^2.
15. State and prove Crammer Rao Inequality.
16. Let $x_1, x_2, \ldots\ x_n$ be a random sample from a population. Prove that the sample mean is an unbiased estimator of the population.
17. Obtain the estimators of μ *and* σ^2 by the method of moments.

18. If t_n is a consistent estimator of θ then show that t_n^2 is also a consistent estimator of θ^2.
19. If t_n is an unbiased estimator of θ then show that t_n^2 is also a unbiased estimator of θ^2.
20. Let $x_1, x_2, \ldots\ x_n$ be a random sample from a normal population with μ and variance σ^2. Show that the sample mean is a sufficient estimator of the population mean.
21. Show that the sample mean is both unbiased and sufficient estimator of λ in the case of Poisson population.
22. Explain the test procedure for testing the mean of two normal populations when variances are unknown and unequal.
23. Explain the method of moments with an example.
24. Explain the properties of an estimator.
25. Show that the sample mean is both unbiased and consistent estimator for the population mean of a normal population.
26. Obtain the estimator for the parameters μ and σ^2 by the method of moments in case of normal distribution.

SECTION – C

27. State and prove Crammer Rao Inequality.
28. Explain the different types of estimator with an example.
29. What is sufficient estimator? For the normal population with mean θ and variance unity show that sample mean is unbiased, consistent, efficient and sufficient for θ.
30. For the distribution with density function $f(x,\theta) = (1+\theta)x^{\theta}$, $0 < x < 1$. Obtain the estimator of θ by the method of moments.

Unit – V – Tests of Hypothesis

SECTION -A

1. What are Type I and Type II error?
2. Define null and alternative hypothesis.
3. Define simple and composite hypothesis.
4. What do you mean by likelihood ratio test?
5. Define the size and power of a test.

SECTION - B

6. Explain the most powerful test.
7. If X ≤ 0.25 is a critical region for testing $H_0 : \theta = 2$ against the alternative hypothesis
   $H_1 : \theta = 3$, on the basis on the single observation from the population
   $f(x, \theta) = (1+\theta)x^{\theta}, 0 \le x < 1$. Obtain the value of Type I error and Type II error.
8. Explain Likelihood Ratio test.

SECTION – C

9. Explain Neymann Pearson lemma.
10. Test the hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta = \theta_1 < \theta_0$.For the probability density function $f(x, \theta) = \theta e^{-\theta x}, 0 < x < \infty$.
11. If X ≥ 1 is a critical region for testing $H_0 : \theta = 2$ against the alternative hypothesis
    $H_1 : \theta = 1$, on the basis on the single observation from the population
    $f(x, \theta) = \theta e^{-\theta x}, 0 < x < \infty$. Obtain the value of Type I error and Type II error and power of the test.
12. Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$, the coin is tossed 5 times . H_0 is rejected if more than 3 heads appeared. Find the probability of type I error and the power of the test.
13. If the density $f(x, \theta) = \begin{cases} 1/\theta \\ 0 \ , \ otherwise \end{cases}, 0 < x < \infty$ and if you want to test $H_0 : \theta = 3$ against $H_1 : \theta = 2$ by means of a single observation.

i. What would be the size of Type I error and Type II error.

If you choose the interval x < 1 as the critical region

ii. What would be the size of the error if 1 < x < 2 as the critical region.

14. Given the density function $f(x,\theta) = \begin{cases} \frac{1}{\theta} & , 0 < x < \infty \\ 0 , & otherwise \end{cases}$ and if you want to test H$_0$: θ = 1 against H$_1$: θ = 2 by means of a single observation. Find the Type I error and Type II error. If the critical region is

i. 0.5 < x

ii. 1 ≤ x ≤ 1.5. Also obtain the power of the test.

15. If X ≤ 0.25 is a critical region for testing H$_0$: θ = 2 against the alternative hypothesis

H$_1$: θ = 3, on the basis on the single observation from the population $f(x,\theta) = (1+\theta)x^{\theta}, 0 \le x < 1$. Obtain the value of Type I error and Type II error.

16. Find the critical region of the likelihood test for testing the null hypothesis H$_0$: $\mu = \mu_0$ against the composite alternative hypothesis H$_0$: $\mu \neq \mu_0$ on the basis of the random sample of size n from a normal population with the known variance σ^2.