D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1.

II- M.Sc. MATHEMATICS

SUBJECT: NUMERICAL ANALYSIS **SUBJECT CODE: 15CPMA3E**

UNIT-I

Finite digit arithmetic and errors and non- linear equation

SECTION-A **6 Marks**

1. Associative law is not true in the case of finite digit arithmetic
   a = 0.345×10^{0}, b = 0.245×10^{-3}, c = 0.432×10^{-3} use 3 digit arithmetic with rounding.
2. Compute 403000×0.0197 by 3 digit arithmetic with rounding.
3. Prove that $\frac{a-b}{c} \neq \frac{a}{c} - \frac{b}{c}$, a = 0.41, b = 0.36, c = 0.70 use 2 digit arithmetic with rounding.
4. Find propagated relation error in addition and subtraction prove that
   $$x_{r+y} = r_x \cdot \frac{x}{x+y} + r_y \cdot \frac{y}{x+y}$$
   $$x_{r-y} = r_x \cdot \frac{x}{x-y} + r_y \cdot \frac{y}{x-y}$$
5. Prove propagative relative error in multiplication prove that $r_{xy} \simeq r_x + r_y$.
6. Prove that $\frac{r_x}{y} \simeq r_x - r_y$.
7. Solve the equation $x^2 + 9.9x - 1 = 0$ by using 2 digit arithmetic with rounding.
8. Evaluate f(x) = $\frac{1}{\sqrt{1+x^2}-\sqrt{1-x^2}}$ for x = 0.2×10^{-1} using 3 digit arithmetic with rounding.
9. Evaluate f(x) = $\frac{1-\cos x}{x}$ for x = 0.01 using 5 dig it decimal arithmetic.
10. Evaluate x = 0.125 in nested form using 5 digit arithmetic floating point with rounding $9.26x^3 - 3.48x^2 + 0.436x - 0.0182$.
11. Find the root of the equation f(x) = $x^3 + x - 1 = 0$ by using bisection method starting with$a_0 = 0$, $b_0 = 0$ and do 5 iterations.
12. Find the root of the equation f(x) = $x^3 - 3x + 1 = 0$ by using scant method taking$x_0 = 1$, $x_1 = 0.5$ and do 3 iterations.
13. Find the root of the equation f(x) = $x^3 - 3x + 1 = 0$ starting with a = 0 and b = 0.5 by using regula falsi method.
14. Explain about the bisection method.

15. Explain about the secant method.
16. Explain about the Newton's method.
17. Explain about the Muller's method.
18. Find the root of the equation f(x) = $x^3 - 3x + 1 = 0$ by using Newton's method with $x_0 = 0.5$ and do 5 iteration.
19. Find cubic root of 'a' by using Newton's method.

SECTION-B 15 marks

1. Let f(x) be n$-\beta$ floating point representation of the real number x then
   i. $|r_x| < \frac{1}{2}\beta^{-n+1}$ (if rounding is used)
   ii. $0 \leq r_x \leq \beta^{-n+1}$ (if chopping is used)
2. Find the root of the equation f(x) = $x^3 + x - 1 = 0$ by using bisection method starting with $a_0 = 0, b_0 = 1$
3. Let f(x) = = $x^3 - 2x^2 + x - 2 = 0$ do 5 iterations to find the root of the equation using secant method with $x_0 = 1.25, x_1 = 1.5$
4. Find the root of the equation $x^3 - 3x + 1 = 0$ by using Muller's method with $x_0 = 0.5, x_2 = 0, x_1 = 1$ (do two iterations).

UNIT-II System of linear equations

SECTION-A 6marks

1. Solve the equations by using simple pivoting
   $0x_1 + x_2 + x_3 = 1$
   $x_1 + 0x_2 + x_3 = 1$
   $x_1 + x_2 + 0x_3 = 1$
2. Solve the system of given equation by using Gauss elimination with scaling, partial pivoting and 5 digit decimal arithmetic with rounding store the multipliers also
   0.4x+8y+5z = −8.2
   6x+0.5y+0z = −19.5
   5x−3y+0.2z = 19.6
3. Solve the matrix by using Gauss elimination with scaling and partial pivoting and 5 digit arithmetic with rounding

$$A = \begin{pmatrix} 0.4 & 8 & 5 \\ 6 & 0.5 & -10 \\ 5 & -3 & 0.2 \end{pmatrix}$$

4. Solve the following system of equation by crout's method using 5 digit arithmetic

   $x_1 + x_2 - 2x_3 = 2.5$

   $4x_1 - 2x_2 + x_3 = 5.5$

   $3x_1 - x_2 + 3x_3 = 9$

5. Solve the following system of equation by crout's method using 5 digit arithmetic

   $x_1 + 2x_2 + x_3 = 2$

   $2x_1 + x_2 - 10x_3 = 4$

   $2x_1 + 3x_2 - x_3 = 2$

6. Find the inverse of the matrix by using Gauss elimination and 5 digit arithmetic, also find $A = \begin{bmatrix} 1 & 1 & -2 \\ 4 & -2 & 1 \\ 3 & -1 & 3 \end{bmatrix}$ the value of $|A|$

7. Find the inverse of matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ by crout's method.

8. Explain about the Ill condition system.

9. Let AX = b be written as X = BX+C with some $\|B\| < 1$, then X = BX+C has unique solution further the sequence $\{x^n\}$ generated by $x^{(m+1)} = Bx^{(m)} + C$, starting with some initial value $x^{(0)}$ will converge to true solution vectorξ.

10. Derive the inequality to find the minimum number of iterations required to meet certain accuracy of the solution is derived.

SECTION-B 15 Marks

1. Let AX = b be the system of equation with

   $A = \begin{bmatrix} 0.003 & 4.00 & 5.00 \\ -3.00 & 3.85 & -6.75 \\ 4.75 & -5.25 & -3.50 \end{bmatrix}$ $b = \begin{bmatrix} 9.003 \\ -5.900 \\ -4.750 \end{bmatrix}$

   Gauss elimination is used with 5 digit arithmetic with rounding.

2. Solve the following system of equation by using crout's method

   $6x_1 + 3x_2 + x_3 = 12$

   $2x_1 + 5x_2 + 2x_3 = 3$

   $2x_1 + 4x_2 + 7x_3 = 21$

3. Find the inverse of the matrix by using Gauss elimination $A = \begin{bmatrix} 4 & 0.5 & 2 \\ 0.6 & -3 & 4 \\ -5 & 2 & 0.8 \end{bmatrix}$

   also find the determinant value of $|A|$

4. Find the inverse of the matrix by using crout's method $\begin{bmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{bmatrix}$

5. Solve the system of equation by using Jacobi's method using 5 digit arithmetic also find the minimum number of iterations required so that the solution is correct to 6 decimal places

   $3x - 6y - 2z = 23$

   $-4x + y - z = -15$

   $x - 3y + 7z = 16$

UNIT-III INTERPOLATION SECTION-A 6 Marks

1. Explain about the lagrangian interpolating polynomial.
2. Find the interpolating polynomial in lagrangian form for the given data

x	−2	−1	1	3
f(x)	−15	−4	0	20

3. Find the interpolating polynomial is lagrangian form for the data

x	−2	0	1	3
f(x)	7	3	1	27

4. Find the divided difference form of polynomial for the given data

x	−2	−1	1	3	4
f(x)	−15	−4	0	20	−39

5. Find the divided difference form of polynomial for the given data

x	−3	−2	0	1	2	4
f	23	−10	−4	−1	18	296

hence interpolate f(−1)

SECTION-B 15 marks

1. Generate the forward difference table and find interpolating polynomial for the data

x	0	0.2	0.4	0.6	0.8
f	0.12	0.46	0.74	0.9	1.2

hence interpolating the value of f(0.1)

2. Prepare the forward difference table for the data

x	−1	0	1	2	3
f	10	2	0	10	62

find the approximate value of f(−0.5)

3. Prepare the forward form of interpolating polynomial for the data

x	2	2.2	2.4	2.6	2.8	3
f	0.301	0.342	0.380	0.415	0.447	0.477

hence estimate f(2.15)

4. Find the given data

x	0.2	0.4	0.6	0.8	1.0
f	3.2	3.6	2.8	3	2.4

by using backward difference of polynomial hence interpolate f(0.95)

5. Find the data

x	0	2	4	6	8
f	1	5	31	121	341

hence interpolate f(7) by using backward difference form

6. Find the given data

x	0	0.2	0.4	0.6	0.8
f	0.12	0.46	0.74	0.9	1.2

by using backward difference form and hence interpolate at x = 0.65

7. Find hermit interpolating polynomial for the data f(1) = −4, $f'(-1) = 12$, f(0) = −1, $f'(0) = 0$, f(2) = 23, $f'(2) = 60$

8. Find the oscillatory interpolating polynomial for the given data f(−1) = −2, $f'(-1) = 13$, f(0) = 3, $f'(0) = 0$, $f''(0) = -8$, f(2) = 19 hence interpolate f(0.5)

9. Find the divided difference form of polynomial for the given data

x	−3	−2	−1	0	2
f	−239	−29	1	1	31

Add f(3) = 241 and f(4) = 104 to the table prepared using the table, after find the polynomial, which interpolates f(x) at −1, 0, 2 and 3. Hence interpolate f(−0.5)

UNIT- IV NUMERICAL DIFFERENTIATION

SECTION-A 6 Marks

1. Find first derivative of f(x) at

x	0.1	0.2	0.3	0.4	0.5	0.6
f(x)	0.425	0.475	0.400	0.450	0.525	0.575

2. Explain the basic trapezoidal rule.
3. Explain the compute trapezoidal rule.
4. Use compute trapezoidal rule to evaluate the integral $\int_{0.1}^{0.6} f(x)dx$ of the function f(x) is given by

x	0.1	0.2	0.3	0.4	0.5	0.6
f(x)	0.425	0.475	0.4	0.450	0.575	0.675

5. Evaluate the integral $\int_{1}^{2} \frac{e^{2x}}{1+x^2}$ using composite trapezoidal with 6 function values that is h = 0.2
6. Evaluate the integral $\int_{-1}^{1} x^2 e^{-x}$ by composite simpsons $\frac{1}{3}$ rule h = 0.25
7. Using composite $\frac{1}{8}$ rule, evaluate $\int_{1}^{3} f(x)dx$ for the data

x	−1	−0.5	0	0.5	1	1.5	2	2.5	3
f(x)	7	5	3.5	4	5.5	6	65	5	4.5

8. Using composite Simpson's $\frac{3}{8}$ rule find the velocity after 18 sec. If the rocket has acceleration as given in the table

t (time in	0	2	4	6	8	10	12	14	16	18

sec)										
a (m/s)	40	60	70	75	80	83	85	87	88	88

9. Using simpsons $\frac{1}{3}$ rule, evaluate $\int_{-1}^{3} f(x)dx$ for the data

x	−1	0.5	0	0.5	1	1.5	2	2.5	3
f(x)	7	5	3.5	4	5.5	6	6.5	5	4.5

10. Use composite trapezoidal rule to evaluate $\int_{0}^{2} \frac{e^{-x^2}}{1+x} dx$ with sparing h = 0.25

11. Evaluate the $\int_{0}^{2} \frac{e^{-x^2}}{1+x} dx$ with h = 0.25 using composite Simpsons $\frac{1}{3}$ rule.

12. A rocket is launched given below a table of acceleration 'a ' at seconds, find the velocity after 90 seconds using Simpson's $\frac{3}{8}$ rule

t	0	10	20	30	40	50	60	70	80	90
a	30	35	40	50	60	75	90	95	105	120

13. Evaluate the $\int_{1}^{4} \frac{xe^{2x}}{1+x^2} dx$ using composite simpsons $\frac{3}{8}$ rule with h = 0.3

14. Explain about the method of undeterminant parameters.

15. Evaluate $\int_{-1}^{1} \frac{x \sin x}{1+x^2} dx$ using 3 point gauss Legendre quadrature.

16. Evaluate $\int_{-1}^{1} \frac{(1+x)e^{x}}{\sqrt{1-x^2}} dx$ using 3 point Gaussian quadrature.

17. Evaluate the integral $\int_{-\infty}^{\infty} \frac{e^{-x^2}}{1+x^2} dx$

18. Evaluate $\int_{0}^{\infty} \frac{e^{-x} \sin 2x}{1+x^2} dx$ using 2 point gauss Legendre quadrature.

SECTION-B 15 Marks

1. Find A_0, A_2, x_1 so that the following rule exact for all polynomial of degree as high as possible and find the error term also $\int_0^2 f(x)dx = A_0 f_0 + \frac{4}{3} f_1 + A_1 f_2 + E(I)$ where E(I) = $\alpha f'''(\xi)$
2. Find x_0, x_1, A_0, A_1 ξ α so that the following rule is exact for all polynomial of degree as high as possible $\int_{-1}^{1} f(x)dx = A_0 f_0 + A_1 f_1 + \alpha f^{IV}(\xi)$
3. Derive the formula for 2 point gauss Legendre quadrature.
4. Derive the formula for 3 point gauss Legendre quadrature.
5. Derive the formula for 2 point and 3 point gauss chebychev quadrature.
6. Derive the formula for 2 point gauss hermit quadrature.
7. Derive the formula for 3 point gauss hermit quadrature.
8. Prove that $\int_0^\infty e^{-x^2} f(x)dx = \frac{2+\sqrt{2}}{4} f(2-\sqrt{2}) + \frac{2-\sqrt{2}}{4} f(2+\sqrt{2}) + E(I)$ by using gauss leguere quadrature.
9. Evaluate the integral $\int_{-1}^{2}\int_{1}^{3} (x^2+y^2)dydx$ using composite trapezoidal with spacing h = k = 0.5 along x-axis and y-axis.

UNIT-V Difference equation SECTION-A 6 Marks

1. Find the solution of homogenous difference equation.
2. Solve $y_{n+2} - 5y_{n+1} + 6y_n = 4$
3. Solve $y_{n+2} - 4y_{n+1} + 3y_n = 2^n \quad y_0 = 1, y_1 = 1$
4. Solve $y_{n+2} - 3y_{n+1} + 2y_n = 2^n$
5. Solve $y_{n+2} + 4y_n = 0$
6. Solve $y_{n+2} - 3y_{n+2} + 3y_{n+1} - y_n = 4n + 3$
7. Solve $y_{n+2} - 4y_{n+1} + 4y_n = 8n$
8. Explain about the single step method.
9. Solve $\frac{dy}{dx} = xy^2 + e^x$, y(1) = 4 spacing h = 0.1 by Taylor's series method.
10. Do 2 step method of Runge-kutta method for the equation $\frac{dy}{dx} = xy + y^2$ y(1) = 2 spacing h = 0.1
11. Explain about the Adam-bash fork method.

12. Explain about the Milni method
13. Explain about Adam-Moulton method.

SECTION-B 15 Marks

1. Solve the difference equation obtain by applying Euler's method to the equation

   $\frac{dy}{dx}$ = 2y y(0) = 1 also find the value of y with spacing h = 0.1 over the interval (0 1)

2. Compute approximation to y(2.1) and y(2.2) as a solution of difference equation

   $\frac{dy}{dx} = x^2 + y^2$ y(2) = 3 using 4th order Runge-Kutta method.

3. Compute the value of y at x = 1.4 as a solution of $\frac{dy}{dx} = xy + x^2 - 1$ with spacing

   h = 0.1, $x_0 = 1, x_1 = 1.1, x_2 = 1.2, x_3 = 1.3$ and $y_0 = 0.649, y_1 = 0.731,$ $y_2 = 0.854, y_3 = 1.028$ using adom bashfourth predictor corrector method.

4. Use second order Runge-Kutta method to compute the values of y(0.1) and z(0.1) as solution of $\frac{dy}{dx} = y + z = f_1$(x,y,z), y(0) = 0, h = 0.1

   $\frac{dz}{dx} = -yz = f_2(x, y, z)$, z(0) = 1

5. Solve by second order Runge-Kutta method

   $\frac{dy}{dx} = y + \frac{z}{2} - x = f_1(x, y, z)$, hence h = 0.1, y(0) = 1

   $\frac{dz}{dx} = \frac{1}{2}y + z + x = f_2(x, y, z)$, z(0) = 1

6. Solve the initial value problem

   $\frac{d^2y}{dx^2} + 4y = x$ y(0) = 1, $y'(0) = 2.25$, h = 0.1