D.K.M COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1.

DEPARTMENT OF MATHEMATICS

ELECTIVE: OPERATION RESEARCH-15CMA6E

III B.SC., MATHEMATICS

UNIT –I GAME THEORY 2 MARKS

1. Define pure and mixed strategy
2. What is the saddle point of $\begin{pmatrix} -4 & 3 \\ 3 & 7 \end{pmatrix}$
3. What is the value of the game whose payoff matrix to A is given below $\begin{pmatrix} 10 & 7 \\ 8 & 9 \end{pmatrix}$
4. When do you say a game is stable?
5. What is meant by minimax, maximin?
6. What is meant by pay-off marix?
7. Define two person zero-sum game
8. What is meant by strategy?
9. Define saddle point with reference to a game
10. State the rules of dominance
11. What do you understand by a value of the game?
12. Check whether game is fair $\begin{pmatrix} 1 & 1 \\ 4 & -3 \end{pmatrix}$
13. Write down the formula for determining the mixed strategy and the value of the game for a 2x2 game
14. Define optimal stratiges, Rectangular game
15. Write any two characteristics of game

5 MARKS

1. What are the main characteristics of game theory?
2. For the following pay-off matrix for firm A, determine the optimal strategies for the both the firms & find the value of the game

Firm B

Firm A					
	3	-1	4	6	7
	-1	8	2	4	12
	16	8	6	14	12
	1	11	-4	2	1

3. Solve the following 2X2 game using algebraic method $\begin{pmatrix} 2 & 5 \\ 7 & 3 \end{pmatrix}$
4. Using principle of dominance to solve the following game

Player B

Player A				
	8	10	9	14
	10	11	8	12
	13	12	14	13

5. For what value of λ, the game with the following matrix is strictly determinable

Player B

Player A			
	λ	6	2
	-1	λ	-7
	-2	4	λ

6. For the game with pay-off matrix

Player B

Player A

Determine the	-1	2	-2
	6	4	-6

best strategies for players A & B also the value of the game. Is this game (i) fair (ii) strictly determinable

7. What are the main characteristics of game theory?
8. Solve the following 2x2 game without saddle point

Player B

Player A		
	6	-3
	-3	0

9. Solve the following game by Arithmetic method $\begin{pmatrix} 6 & -3 \\ -3 & 0 \end{pmatrix}$
10. A and B play a game in which each has 3 coins, 5p, a 10p and a 20p. Each selects a coin without the knowledge of other's choice. If the sum of the coins is an odd amount, A win's B's coin: If the sum is even B win's A's coin. Find the best strategy for each player and the value of the game

TEN MARKS QUESTIONS

1. Use dominance property to solve the following game

Player B

Player A

	B_1	B_2	B_3	B_4
A_1	21	8	9	7
A_2	9	5	16	8
A_3	14	10	20	6
A_4	10	9	15	1

2. Solve the following 2X5 game by graphic method:

Player B

Player A

	1	2	3	4	5
1	-5	5	0	-1	8
2	8	-4	1	6	-5

3. Use the notion of dominance to simplify the rectangular game with the following pay-off & solve it by Graphic method

Player K

Player L	I	II	III	IV
1	18	4	6	4
2	6	2	13	7
3	11	5	17	3
4	7	6	12	2

4. Solve the following game $\begin{pmatrix} 3 & -1 & 1 & 2 \\ -2 & 3 & 2 & 3 \\ 2 & -2 & -1 & 1 \end{pmatrix}$

UNIT II – INTEGER PROGRAMMING 2 MARKS

1. What is integer programming?
2. Define pure and mixed integer programming problems
3. Differentiate between pure and mixed integer programming problems
4. Give any two applications of integer programming
5. What are the methods of solving an integer programming problem?
6. What is the fractional part of $-\frac{2}{3}$?
7. Write down the Gomory's fractional cut corresponding to the equation $x_1 + \frac{2}{3}x_3 - \frac{1}{3}x_4 = \frac{2}{3}$ that appears in the non-integer optimal simplex table of an integer programming problem
8. What is Gomory constant that is inclined in the simplex table is otherwise known?
9. What is the integer part of -22 / 5?
10. Can the mixed cut eventually produce the optimum integer solution of a problem in which all variables are integers?

SECTION-B 5 MARKS

1. Explain the imp**o**rtance of integer programming problems and their applications
2. What are the applications of integer programming problem?
3. Explain whether an IPP can be solved by rounding off the corresponding simplex table
4. Write a brief note on cutting plane method
5. Explain Gomory's mixed IPP & its algorithm

SECTION-C 10 MARKS

1. Describe Gomory's fractional cut algorithm
2. Find the optimum integer solution to the following linear programming problem

   Max $z = x_1 + x_2$

   Subject to $3x_1 + 2x_2 \le 5$

$x_2 \le 2$ & $x_1,\ x_2 \ge 0$ and are integers

3. What are the applications of integer programming problem?
4. Solve the following IPP
   Min $z = -2x_1 - 3x_2$
   Subject to $2x_1 + 2x_2 \le 7$
   $x_1 \le 2$, $x_2 \le 2$ & x_1, $x_2 \ge 0$ and are integers
5. Find the optimum integer solution to the following LPP
   Max $z = 4x_1 + 6x_2 + 2x_3$
   Subject to $4x_1 - 4x_2 \le 5$, $-x_1 + 6x_2 \le 5$, $-x_1 + x_2 + x_3 \le 5$
   x_1, x_2, $x_3 \ge 0$, x_1, x_3 integers

UNIT III – QUEUEING THEORY SECTION-A 2 MARKS

1. Define a Queue
2. What are the basic characteristics of a queue system?
3. Define Transient state, Steady state
4. What do you understand by Kendal's Notation?
5. Define Utilisation factor in Queueing theory
6. Give the formula for probability of *n* units in the system under single server, FCFS discipline
7. Give an example of first come, last served
8. Give the formula for length of system under (M/M/1: N/FCFS)
9. What do you understand by term balking?
10. State the components of a queue
11. Define M/M/1 queueing system
12. The interarrival time under queue follows which distribution
13. At a public telephone booth the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone, what is the expected number of callers in the booth at any time?
14. How do you describe queueing system?

SECTION-B 5 MARKS

1. Customers arrive at a box office window, being manufactured single individual according to Poisson input process with mean rate of 30 per hour. The time required to serve a customer follows exponential with a mean of 90 seconds. What is the average waiting time of a customer?
2. In a public telephone booth the arrivals are on an average 15 per hour. A call on an average takes 3 minutes follows exponential. If there is just one phone, find (a) Expected number of callers in the booth at any time. (b) The proportion of the time the booth is expected to be idle
3. If for a period of 2 hours in a day (8 – 10 AM) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes then calculate for this period (a) The Probability that the yard is empty (b) Average queue length, assuming that capacity of the yard is 4 trains only

4. In a railway marshalling yard, goods trains arrive at the rate of 50 trains per day. Assume that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes. Calculate (a) The probability that the yard is empty (b) Average queue length
5. Customers arrive at the first class, ticket counter of a theatre at a rate of 12 per hour. There is one clerk servicing the customers at the rate of 30 per hour. (i) What is the probability that there is no customer at the customer? (ii)What is the probability that there are more than 2 customer at the counter?

SECTION-C 10 MARKS

1. A T.V repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day. What is repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?
2. At a one-man barber shop the customers arrive following Poisson process at an average rate of 5 per hours and they served according to exponential distribution with an average service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customer spends in the system
3. Cars arrive at a petrol pump, having one petrol unit, in Poisson fashion with an average of 10 cars per hour. The service time is distributed exponentially with a mean of 3 minutes. Find (a) Average number of cars in the system (b) Average waiting time in the queue (c) Average queue length (d) The probability that the number of cars in the system is 2
4. At a public telephone booth in a post office arrivals are considered to be poisson with an average inter-arrival time of 12 minutes. The length of the phone call may be assumed to be distributed exponentially with an average of 4 minutes. Calculate the following
   (i) What is the probability that a fresh arrival will not have to wait for the phone?
   (ii) What is the probability that an arrival will have to wait more than 10 mins before the phone is free?
   (iii) What is the average length of queue formed from time to time?

UNIT IV – SEQUENCING PROBLEM 2 MARKS

1. Define a Sequencing problem
2. Write any two assumptions made in a sequencing problem
3. Define No passing rule in sequencing problem
4. Define total elapsed time & idle time on machines
5. What is the algorithm used to find the optimal sequence of n jobs through two or three machines
6. What is the objective of sequencing problem?
7. What do you meant by job sequence?

8. Write the condition for Johnson's algorithm to be applicable in finding the optimal sequencing order of n jobs through 3 machines
9. If t11 = 4, t21 = 8, t31 = 5, t12 = 6, t22 = 3 and t32 = 7, where tij is the time required to process the ith job in the jth machine, find the optimal sequence of jobs so as to minimize the total processing time.
10. What are the assumptions made to convert a 3 machine problem to a 2 machine problem?

SECTION-B 5 MARKS

1. Determine the sequence of 5 jobs that will minimize the elapsed time T

Job	1	2	3	4	5	6	7
Machine A	3	12	15	6	10	11	9
Machine B	8	10	10	6	12	1	3

2. Write the procedure to determine an optimal sequence for n jobs on two machines
3. Determine the total elapsed time and ideal times for the 5 jobs through 3 machines whose sequence 4-1-5-2-3 and the processing time in hours are given below

Job	1	2	3	4	5
Machine A	3	8	7	5	4
Machine B	4	5	1	2	3
Machine C	7	9	5	6	10

4. Seven jobs are to be completed by machining through 2 machines. The time in minutes taken for each of these jobs at different machines are given below. Find an order in which the jobs are to be completed so as to minimize the total elapsed time

Job	1	2	3	4	5	6	7
Machine A	5	7	3	4	6	7	12
Machine B	2	6	7	5	9	5	8

SECTION-C 10 MARKS

1. We have Seven jobs, each of which must go through machines A,B and C in the order A-B-C. Processing times in hours are given in the following table. Determine the optimal Sequence of jobs that minimizes the total elapsed time, idle time of machines A, B and C

Job	1	2	3	4	5	6	7
Machine A	3	8	7	4	9	8	7
Machine B	4	3	2	5	1	4	3
Machine C	6	7	5	11	5	6	12

2. Two jobs are processed on 4 machines A, B, C & D the technological order of these jobs of processing times are as follows:
   Job 1: A B C D
   Job 2: D B A C
   Processing times are given in the following table

Machine:	A	B	C	D
Job 1:	4	6	7	3
Job 2:	4	7	5	8. Find the total elapsed time

3. Determine the minimum time needed to process the two jobs on 4 machines M_1, M_2, M_3& M_4. The technological order of these jobs is as given below. Use graphical method
   Job 1: M_1 M_2 M_3 M_4
   Job 2: M_4 M_2 M_1 M_3
   Processing times are given in the following table

Machine:	M_1	M_2	M_3	M_4
Job 1:	5	7	8	4
Job 2:	5	8	6	9

4. Explain the Johnsosn's algorithm of solving a sequencing problem of n jobs on 2 and 3 machines.
5. Explain a sequencing problem

UNIT V – NETWORK ANALYSIS SECTION-A 2 MARKS

1. What is dummy activity?
2. Define optimistic time estimate
3. What is a network?
4. Define critical path
5. Define optimistic and pessimistic time
6. Write expansion of PERT
7. Define setup cost
8. Define Lead time
9. Define total, free and independence float
10. Define network diagram
11. Define project network
12. What are the three main phases of a project?
13. What are the rules for constructing a project network?
14. What is dangling of a network?
15. Define total float of an activity

SECTION-B 5 MARKS

1. What are rules for construction of network?
2. What is the significance of a dummy activity?
3. Explain independence float
4. What is the significance of float in CPM?
5. Draw the network for the following
   B > A; C > B; D, E > C; F > D; G > E; H > C
6. Distinguish between PERT and CPM
7. A project has the following characteristics:

Activity:	1-2	2-3	2-4	3-5	4-5	4-6	5-7	6-7	7-8	7-9	8-10	9-10
t_o:	1	1	3	2	3	4	6	2	4	1	3	
t_m:	2	3	4	3	5	5	7	4	6	2	5	
t_p:	3	5	5	4	7	6	8	6	8	3	7	

   (i) Construct the network
   (ii) Find the expected duration and the variance of each activity
8. Construct the network for the project whose precedence relationship are as given below

   B < E, F; F,C< G; E,G < H; L,H < I; L < M; H ,M < N; H < J; I ,J < P; P < Q
9. A project consists of 12 jobs

Jobs:	1-2	2-3	2-4	3-4	3-5	4-6	5-8	6-7	6-10	7-9	8-9	9-10
Duration:	2	7	3	3	5	3	5	8	4	4	1	7

   Draw a project network and determine the critical path and project duration
10. Distinguish between earliest expected time and latest expected time.

SECTION-C 10 MARKS

1. Construct the network for the project whose activities are given below and compute the total, free,

Independence float of each activity and hence determine the critical path and the project duration

Activity:	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration:	38	12	6	3	3	8	5	3	8	

2. A project consists of the following activities and time estimates

Activity	Least time(days)	Greatest time(days)	Most likely time(days)
1-2	3	15	6
2-3	2	14	5
1-4	6	30	12
2-5	2	8	5
2-6	5	17	11
3-6	3	15	6
4-7	3	27	9
5-7	1	7	4
6-7	2	8	5

(i) Draw a network
(ii) What is the probability that the project will be completed in 27 days?

3. A project has the following properties. Find the critical path

Activity:	1-2	1-3	2-4	2-5	3-4	4-5
Duration:	8	4	10	2	5	3

4. Following table gives activities and the different time durations ($t_\circ, t_m, t_p$)

Activity:	1-2	1-3	1-4	2-5	3-5	4-6	5-6
$t_\circ$	5	1	2	3	1	2	1
t_m	6	1	4	6	1	2	4
t_p	7	7	12	15	1	8	7

5. Consider the following data for the activities concerning a project:

Activity:	A	B	C	D	E	F
Immediate Predecessor:	-	A	A	B,C	-	E
Duration(days)	2	3	4	6	2	8

(i) Draw the network diagram
(ii) Find the minimum project completion time
(iii) Determine the critical path
(iv) Compute earliest event time and latest event time

6. The following table shows the jobs of a network along with their time setimates

Job:	1-2	1-6	2-3	2-4	3-5	4-5	6-5	5-8	7-8
a (days):	1	2	2	2	7	5	5	3	8
m (days):	7	5	14	5	10	5	8	3	17
b (days):	13	14	26	8	19	17	29	9	32

Draw the project network and find the probability that the project completed in 40 days

7. The following table list the jobs of a network along with their time estimates

Jobs	1-2	1-3	2-4	3-4	4-5	3-5
Optimistic time	2	9	5	2	6	8
Most likely time	5	12	14	5	6	17
Pessimistic time	14	15	17	8	12	20

(i) Draw the network
(ii) Find the probability that the project will be completed within 30 days?
(iii) Calculate the length and variance of the critical path.