D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1.

I M.Sc MATHEMATICS

UNIT-I ORDINARY DIFFERENTIAL EQUATION

1. Write about linear equation with constant co-efficient.
2. Write about homogenous and non-homogenous equation.
3. Define differential operator.
4. Define second order homogenous equation.
5. Define characteristic polynomial and its different cases.
6. If ϕ_1, ϕ_2 be any two solution of L(y)=0 and c_1,c_2 are two constants then the function $\phi=c_1\phi_1+c_2\phi_2$ is also a solution of L(y)=0.
7. Find the solution of the differential equation $y^{11}+y^{1}-2y=0$.
8. Define initial value problem for second order equation.
9. Define the solution of the initial value problem $y^{11}-2y^{1}-3y=0$, y(0)=0, $y^{1}(0)=1$.
10. Define linear dependent and linear independent.
11. Prove that the function defined by $\phi_1=e^{r_1 x}$, $\phi_2=e^{r_2 x}$ are linearly independent on any interval I.
12. Prove that the function defined by $\phi_1=e^{r_1 x}$, $\phi_2=xe^{r_2 x}$ are linearly independent on any interval I.
13. Prove that the function $\phi_1(x)=e^{\alpha x}$, $\phi_2(x)=e^{\alpha(1+x)}$ where $-\alpha<x<\infty$ and α is a constant.
14. Define wronskian of order two.
15. If the determinant Δ of the coefficient in the system of equations.

$a_{11}z_1+a_{12}z_2+\dots\dots\dots+a_{1n}z_n=c_1$

$a_{21}z_1+a_{22}z_2+\dots\dots\dots+a_{2n}z_n=c_2$

....

.... (A)

$a_{n1}z_1+a_{n2}z_2+\dots\dots\dots+a_{nn}z_n=c_n$

where a_{ij} , c_j are complex constant.

I) if $\Delta\neq0$,there is a unique solution of the system for $z_1,z_2,\dots..z_n$ it is given by $z_k=\dfrac{\Delta_k}{\Delta}$ (k=1,2,.......n) where Δ_k is the determinant obtained from Δ by replacing its k^{th} column ie.,)$a_{1k},a_{2k},\dots\dots a_{nk}$ by $c_1,c_2,\dots\dots.c_n$.

ii)$c_1=c_2=\dots\dots.=c_n=0$ in (A) and the determinant of the coefficient $\Delta=0$ there is a solution of(A) such that not all z_k are zero.

16.obtain two solution ϕ_1,ϕ_2 of L(y)=0 are the linearly independent

On an interval I if and only if $W(\phi_1,\phi_2)\neq0$ for all x in I.

17. Let ϕ_1,ϕ_2 be two solution of L(y)=0 on an interval I and let x_0 be

any point I then ϕ_1,ϕ_2 are linearly independent on I iff

$W(\phi_1,\phi_2)(x_0)\neq 0$.

18. Let ϕ_1,ϕ_2 be two linearly independent solution of L(y)=0 on an Interval I.Every solution ϕ of L(y)=0 can be written uniquely as

$\phi=c_1\phi_1+c_2\phi_2$ where c_1,c_2 are constant.

19. If ϕ_1,ϕ_2 are the solution of L(y)=0 on any interval I containing a point x_0 then $W(\phi_1,\phi_2)(x_0)=e^{-a(x-x_0)}\ W(\phi_1,\phi_2)(x_0)$.

20. Is the function $\phi_1(x)=\sin x$, $\phi_2(x)=e^{ix}$ linearly independent?

UNIT-II

21. Define homogenous equation of order n.

22. Let $r_1,r_2,\ldots\ldots.r_n$ be the distinct root of the characteristic polynomial P and Suppose r_i as multiplicity m_i thus $(m_1+m_2+\ldots\ldots..m_s=n)$ then the n function

$$e^{r_1x},xe^{r_1x},\ldots\ldots\ldots\ldots x^{m_1-1}\ e^{r_1x}$$

$$e^{r_2x},xe^{r_2x},\ldots\ldots\ldots\ldots x^{m_2-1}\ e^{r_2x}$$

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$$e^{r_sx},xe^{r_sx},\ldots\ldots\ldots\ldots x^{m_s-1}\ e^{r_sx}$$ are solution of L(y)=0.

23. prove that the n-solution given by

$$e^{r_1x},xe^{r_1x},\ldots\ldots\ldots\ldots x^{m_1-1}\ e^{r_1x}$$

$$e^{r_2x},xe^{r_2x},\ldots\ldots\ldots\ldots x^{m_2-1}\ e^{r_2x}$$

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$e^{r_s x}$,x$e^{r_s x}$,………. $x^{m_s - 1}$ $e^{r_s x}$ are linearly independent on interval I .

24. Define initial value problem for n^{th} order equation.
25. Define wronskian of n derivatives.
26. If $\phi_1, \phi_2, \ldots\ldots\ldots \phi_n$ are n solution of L(y)=0 on any interval I they are linearly Independent if and only if w($\phi_1, \phi_2, \ldots\ldots\ldots \phi_n$)(x)≠0.
27. State and prove existence theorem for order n.
28. let $\phi_1, \phi_2, \ldots\ldots\ldots \phi_n$ be n solution of L(y)=0 on any interval I containing a point x_0 then

    w($\phi_1, \phi_2, \ldots\ldots\ldots \phi_n$)(x)= $e^{-a(x-x_0)}$ w($\phi_1, \phi_2, \ldots\ldots\ldots \phi_n$) ($x_0$).
29. Define non-homogenous equation of order n.
30. write about annihilator method.
31. Using annihilator method find a particular solution of the equation $y^{11}-3y^1+2y=x^2$.
32. write the characteristic polynomial of an annihilator method of function

    (i) e^{ax} (ii) x^k e^{ax} (iii) sinax ,cosax (real)

    (iv) x^k sinax, x^k cosax (a real).
33. Define operators and its different types.
34. The correspondence which associates with each $L=a_0D^n+a_1D^{n-1}+\ldots\ldots\ldots+a_n$ Its characteristic polynomial p is given by $p(r)= a_0r^n+a_1r^{n-1}+\ldots\ldots\ldots+a_n$ is one to one correspondence between all linear differential with constanst coefficient and all polynomials if L,M are associated with p+q, ML i associated pq&

αL is associated with αp (α is a constant).

35. consider the equation with constants coefficient $L(y)=p(x)\, e^{ax}$ where p is a polynomial given by $p(x)= b_0 r^m+b_1 r^{m-1}+........+b_m$ $(b_0\neq 0)$ suppose a is a root of the characteristic polynomial P of L of multiplicity j then there is a unique solution x of equation of the form $X(x)=x^j (c_0 x^m+c_1 x^{m-1}+........+c_m)e^{ax}$ where $c_0,c_1,.........,c_m$ are constants determinant by the annihilator method.

UNIT-III

36. Define linear equation with variable coefficient .

37. Obtain there exist x linear independent solution L(y)=0 on I.

38. Let $\phi_1 ,\phi_2 ,.........\phi_n$ be n solution of L(y)=0 on I satisfying $\phi_i^{(i-1)}(x_0)=1,\ \phi_j^{(j-1)}(x_0)=0,\ i\neq j$ if ϕ is any solution of L(y)=0 on I there are n constant $c_1,.........,c_n$ such that

$\phi = c_1\phi_1+c_2\phi_2+........+c_n\phi_n.$

39. Define linear space.

40. Define basis.

41. Let ϕ_1 be a solution of L(y)=0 on any interval I and suppose $\phi_1(x)\neq 0$ on I if $v_1 ,v_2 ,.........v_n$ is any basis on I for the solution of linear equation

$\phi_1 v^{(n-1)} +.........[n\phi_1^{(n-1)}+(n-1)\ \phi^{(n-2)}+.........+a_{n-1}\phi_1]v=0$ of order

(n-1) & if $v_k= v_k(k=1,2,.......n)$ then $\phi_1 ,u_2\phi_1 ,.........u_n\ \phi_1$ is a basis for the solution L(y)=0 on I.

42. If ϕ_1 is a solution of $L(y)=y^{11}+a_1(x)y^1+a_2(x)=0$ on an interval I

and $\phi_1 \neq 0$ on I. A second solution ϕ_2 of equation on I is given by

$\phi_2, \phi_1(x) = \int_{x_0}^{x} \frac{1}{[\phi_1(x)]^2}$ exp[- $\int_{x_0}^{x} a_1(t)dt$]dx the function ϕ_1, ϕ_2 form a basis for a solution of equation on I.

43. Define legendre equation.

44. Obtain the legendre polynomial of the legendre equation.

45. If $p_n(x)$ is a legendre polynomial of the legendre polynomial then

$$\int_{-1}^{1} p_n^2(x)dx = \frac{2}{2n+1}$$

46. Show that $\int_{-1}^{1} p_n(x)\, p_m(x)dx = 0$ for $m \neq n$ where p_n, p_m are legendre polynomial.

UNIT-IV

47. Prove that a basis for the solution of the Euler equation on any interval not containing x=0 is given by $\phi_1 = |x|^{r_1}$, $\phi_2 = |x|^{r_2}$ In case r_1, r_2 are distinct roots of q and by $\phi_1 = |x|^{r_1}$, $\phi_2 = |x|^{r_1} \log|x|$ if r_1 is a root of multiplicity 2.

48. Find the solution of $x^2y^{11}+xy^1+y=0$ for $x \neq 0$.

49. Let $r_1, r_2, \ldots\ldots r_s$ be the distinct roots of the indicial polynomial q for $L(y) = x^n y^{(n)} + a_1 x^{n-1} y^{n-1} + \ldots\ldots + a_n y = 0 \rightarrow (1)$ and suppose r_i has multiplicity m_i then the function,

$|x|^{r_1}$, $|x|^{r_1} \log|x|$ .... $|x|^{r_1} \log^{m_1-1}|x|$;

$|x|^{r_2}$, $|x|^{r_2} \log|x|$ .... $|x|^{r_2} \log^{m_2-1}|x|$;

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………………

$|x|^{r_s}$, $|x|^{r_s}\log^{|x|}$ ………. $|x|^{r_s}\log^{m_s-1}|x|$;

form a basis for the solution of the n^th^ order Euler equation any

interval not containing x=0.

50. Obtain the second order equation with regular singular point.

UNIT-V

1. Write the general method of successive approximation.
2. State and prove the existence theorem for successive approximation.
3. Find the solution of the equation

   i) $x^2y'' + 2xy' - 6y = 0$

   ii) $2x^2y'' + xy' - y = 0$

4. Obtain the exact equation $M(x,y) + N(x,y)y' = 0$ in rectangle R and F is a real valued function such that $\frac{\partial F}{\partial x} = M, \frac{\partial F}{\partial y} = N$ in R every differential functions ϕ define implicit by a relation $F(x,y) = c$ where C is a constant is the solution of the equation.
5. Write the necessary and sufficient condition for an equation to be exact.
6. Find the solution of the equation $y' = \dfrac{3x^2 - 2xy}{x^2 - 2y}$
7. Solve the equation $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ by the method of successive approximation.
8. Solve the equation $y' = xy, y(0) = 1$ by the method of successive approximation.
9. The successive approximation ϕ_k defined by $\phi_0(x) = y_0$, exist as

   $\phi_{k+1}(x) = y_0 + \int_{x_0}^{x} [f(t)\phi_k(t)]dt$ continuous function on

   $I : |x - x_0| \le \alpha = \min imum\{a, b/m\}$ and $(x, \phi_k(x))$ is in R for x in I indeed the ϕ_k satisfy $|\phi_k(x) - y_0| \le M|x - x_0| \forall x \varepsilon I$
10. Find first four approximation $\phi_0, \phi_1, \phi_2, \phi_3$ to the equation $y' = 3y + 1, y(0) = 2$
11. Show that i) $x^{1/2}J_{1/2}(x)\dfrac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)}\sin x$ ii) $x^{1/2}J_{-1/2}(x)\dfrac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)}\cos x$

12. Let F be a real valued continuous function on the strip $S:|x-x_0|\le a;|y|<\infty$ (a>0) and suppose that f satisfies on S a lipschitz condition with constant k>0.

Find the solution of the equation $x^3y'''-3x^2y''+6xy'-6y=0$ for x>0 and $\phi_1(x)=0$.

UNIT-I Section-B

1. State and prove Existence theorem. .

2. State and prove uniqueness theorem.

3. Let Φ be any solution of L(y)=y$''$+a_1y'+a_2y=0 on an interval I counting a point x_0

then for all x in I.

$\|\Phi(x)\|e^{-k|x-x_0|}\le\|\Phi(x)\|\le\|\Phi(x_0)\|e^{-k|x-x_0|}$ 1

where $\|\Phi(x)\|=\left[|\Phi(x)|^2+\Phi^1(x)^2\right]^{1/2}$ and K=1+$|a_1|+|a_2|$.

4.Let b be continuous on an interval I every solution $\Psi[L(y)]=b(x)$ on I can be written as $\Psi=\Psi_p+C_1\Phi_1+C_2\Phi_2$ Where Ψ_p is a particular solution $\Phi_1 and \Phi_2$ are two linearly independent solution of L(y)=0 and $C_1 and c_2$ are constants.

A particular Solution Ψ_p is given by $\Psi_p(x)=\int_{x_0}^{x}\frac{[\Phi_1(t)\Phi_2(x)-\Phi_1(x)\Phi_2(t)]}{W(\Phi_1,\Phi_2)(T)}dt$

Conversely every such Ψ is a solution of L(y)=b(x) .

5.Solve L(y)=b(x) in the case p(r)=$r^2+a_1r+a_2$ as the two distinct root $\Phi_1(x)=e^{r_1x}$ and $\Phi_2(x)=e^{r_2x}$

UNIT-II

6. obtain the homogenous equation of order n.

7.obtain the n function $\Phi_1\Phi_2\Phi_3.........\Phi_n$ on an interval I are said to be linearly dependent on I if there are constant $C_1,C_2..................C_n$ not all zero such that C_1
$C_1\Phi\ (x)+C_2\Phi_2(x)+..................+C_n\Phi_n(x)=0\vee x\varepsilon I$.

8.Let Φ be any solution of L(y)=y^n +$a_1 y n-1$+$a_2 y n-2$+............+a_n y=0 on an interval Z Containing a$\neq x_0$ then for all $\|\Phi(x_0)\|e^{-k|x-x_0|} \leq \|\Phi(x)\| \leq \|\Phi(x_0)\| e^{k|x-x_0|}$

Where K=1+$|a_1|$ +$|a_2|$ ....$|a_n|$.

9.If $\Phi_1\Phi_2\Phi_3.........\Phi_n$ are n soutions of L(Y)=0 on an interval I they are linearly independent if and only if W($\Phi_1\Phi_2\Phi_3.........\Phi_n$)(x) $\neq 0$.

10.Let $\Phi_1\Phi_2\Phi_3.........\Phi_n$ are linearly independent solution of L(Y)=0 on an interval I if $C_1, C_2..................C_n$ are any constant $\Phi = C_1\Phi\ (x) + C_2\Phi_2(x) + ... + C_n\Phi_n(x)$ is a solution and every solution may be represented in this form.

11.Let If $\Phi_1\Phi_2\Phi_3.........\Phi_n$ are n soutions of L(Y)=0 on an interval I containing a point x_0 then W($\Phi_1\Phi_2\Phi_3.........\Phi_n$)(x) = $e^{-a(x-x_0)}$ W($\Phi_1\Phi_2\Phi_3.........\Phi_n$)($x_0$)

UNIT-III

12. Let b be continuous on an interval Z and let $\Phi_1\Phi_2\Phi_3.........\Phi_n$ be n linearly independent solution of L(y) =0 on I then every solution $\Psi[L(y)] = b(x)$ can be written as $\Psi = \Psi_p + C_1\Phi_1 + C_2\Phi_2 + ... C_n\Phi_n$ where Ψ_p is a particular solution of $[L(y)] = b(x)$ and $C_1, C_2..................C_n$ are constant every such Ψ is a solution of $[L(y)] = b(x)$.

A particular solution Ψ_p is given by $\Psi_p(x) = \sum_{K=1}^{n} \Phi_k(x) \int_{x_0}^{x} \frac{[W_K(t)b(t)]}{W(\Phi_1\Phi_2\Phi_3.........\Phi_n)(t)} dt$.

13.Consider the equation $y''' + y'' + y' + y = 1$ which satisfies
$\Psi(0) = 0, \Psi'(0) = 1, \Psi''(0) = 0$

14.Consider the equation with constant coefficient L(y)=p(x) e^{ax} ....1. Where p is a polynomial given by $p(x) = b_0x^n + b_1x^{n-1} + ... + b_m(b_0 \neq 0)$. Suppose a is a root of the characteristics polynomial p of L of multiplicity j then there is a unique solution Ψ of equation of the form

$\Psi(x) = x^j(C_0x^m + C_1x^{m-1} + ... + C_m(e^{ax})$. where $C_0, C_1, C_2..................C_m$ are constant determine by the annihilator method.

15.State and prove Existence theorem for analytic co efficient.

16.Obtain the solution of Lebseque Equation.

17.Find the Legendre polynomial If $p_n(x)$ is the legendre polynomial of the legendre equation $(1-x^2)y^{''}-2xy^{'}+n(n+1)y=0$ then $p_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$

UNIT-IV

18.State and prove Euler equation.

19.Consider the second order euler equation $x^2y^{''}+axy^{'}+by=0$(a,b constant) and the polynomial q given by q(r)=r(r-1)+ar+b. A basis for the solution of the euler equation on any interval not containing x=0 given by $\Phi_1(x)=|x|^{r_1}$, $\Phi_2(x)=|x|^{r_2}$ incase $r_1 r_2$ are distinct roots of q and by $\Phi_1(x)=|x|^{r_1}$, $\Phi_2(x)=|x|^{r_2}\log|x|$ if r_1 is a root of q of multiplying two.

20.Let r_1, r_2..................r_n be the distinct root of the indicial polynominal q for

$L(y)=x^n y^n + a_1 x^{n-1} + \ldots\ldots\ldots\ldots + a_n y = 0$..............1 and suppose r_i has multiplicity m_i then the function.

$|x|^{r_1}, |x|^{r_1}\log|x| \ldots\ldots\ldots\ldots\ldots\ldots |x|^{r_1}\log^{m_1-1}|x|:$

$|x|^{r_1}, |x|^{r_2}\log|x| \ldots\ldots\ldots\ldots\ldots\ldots |x|^{2}\log^{m_2-1}|x|:$

24.State and prove Existence Theorem.

25. Derive the Bessel function f of zero order of the first kind and second kind.

26. Obtain the solution for the Bessel equation of order α where $\alpha \neq 0$

UNIT-V

27. Write the necessary and sufficient for an equation to be exact.

28. Prove that,

(i) $j^{'}=-j_1$ (ii) $j_2-j_0=2j_0^{''}$ (iii) $j_2=j_0^{''}-(\frac{1}{x})j_0^{'}$

(iv) $j_2+3j_0^{'}+4j_0^{'''}=0$

29.Explain S-machine for successive approximation.

30. State and prove the Existence Theorem for Convergence of the successive approximation and properties of the limit Φ.

31.Let f,g,be continuous on R and suppose f satisfies a Lipchitz Condition there with Lipchitcz Constant K. Let Φ, Ψ be solutions of $y' = f(x,y), y(x_0) = y_1$, $y' = g(x,y), y(x_0) = y_2$

Respectively and show that $\left[f(x,y) - g(x,y)\right] \le t$ and (x,y) in R and $\left|y_1 - y_2\right| \le \delta$ and valid then $\left|\Phi(x) - \Phi(y) \le \delta e^{k|x-x_0|}\right| + \left|e^{k|x-x_0|}\right|$ for all x in I.

32.Compute the first four approximation $\Phi_1, \Phi_2, \Phi_3, \Phi_4$ of $y' = 1 + xy, y(0) = 1$.

33.Compute the wronskian of 4 linearly independent solution of $y^{(4)} + 16y = 0$