D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1.

TENSOR ANALYSIS AND RELATIVITY (UNIT – 1)

SECTION – A (6 MARKS)

1. If a_{pqr} $x^p x^q x^r = 0$ for all values of the independent variables $x^1, x^2 \ldots x^n$ and a_{pqr} are constant. show that $a_{kji} + a_{jki} + a_{ikj} + a_{kij} + a_{jik} = 0$
2. If $a^s{}_r$ is a double system such that arm $a^m{}_s = s^r{}_s$ show that lars1 = ± 1
3. If J and J^1 be the jacobian of the transformation $x^i = \emptyset^i(x^1,x^2\ldots x^n)$ and $\overline{x^i} = 4^i$ $(x^1, x^2\ldots x^n)$ then $JJ^1=1$
4. Prove that there is no distinction betweencontravarient vectors and covariant vectors when we restrict ourselves to orthogonal transformation of rectangular cartosion system of co-ordinates.
5. If $_i$ and μi are the components of a covariant and contravarient vectors respectively the the sum $_I$ μi is invariant.
6. If b^{ij} is reciprocal tensor of a_{ij} then a_{ij} is a reciprocal tensor of b^{ij}
7. Show that $s^i{}_j$ is a mixed tensor of order two.
8. If $a^{ij} u_i u_j = b^{ij} u_i u_j$ for an arbitrary covariant vector u_i, show that $a^{ij} + a^{ji} = b^{ij} + b^{ji}$
9. If aij x aji are reciprocal symmetric tensors of the second order show that
   i. $aij \frac{\sigma aij}{\sigma xk} + aij \frac{\sigma aij}{\sigma xk} = 0$
   ii. $\frac{\sigma \log a}{\sigma xk} = aij \frac{\sigma aij}{\sigma xk} = - aij \frac{\sigma aij}{\sigma xk}$ where

   $a = |aij|$
10. If Aij=0 for i≠j x Aij ≠0 for i=j show that the conjugate tensor Bij=0 for i≠j and Bii=1/Aii

SECTION – B (15 marks)

1. If $a_{ij} A_i B_j = 0$ for two distinct arbitrary vectors A_i and B_j then $a_{ij} = o$
2. Any covariant tensor of second order can be expressed uniquely as the sum of a symmetric x skew – symmetric tensor of second order.
3. If the relation $b^{ij} u_i u_j = 0$ holds for any arbitrary covariant vector u_i, prove that $b^{ij} + b^{ji} = 0$
4. A covariant vector has components xy, 2y- z^2 xz in rectangular Cartesian co—ordinates. Determine its components in spherical polar co-ordinates.
5. If a vector has components x,y in rectangular Cartesian co-ordinates then show that r,θ are components in polar co-ordinates and if a vector has components x y in Cartesian co-ordinates then P.T. its components in polar co-ordinates are r-r θ^2, θ=2/r r θ where dots representing differentiation w.r. to a parameter t.

UNIT – II SECTION – A (6 MARKS)

1. If $ds^2 = (dx)^2 + 2\cos\emptyset\, dxdy + (dy)^2$. Find the co-efficient of Riemam metric.
2. Show that the co-efficient g_{ij} for the Riemann metric is a covariant vector of rank 2.
3. S.T. the no. of independent component of the fundamental metric tensor gij is atmost

n(n-1)/2.

4. Find the conjugate metric tensor in n Riemannian space V_3 in which distance d_s is given by
   $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 3 (dx^1) (dx^2) - 3 (dx^1) (dx^2) - 3 (dx^2) (dx^1) + 2 (dx^2) (dx^3) + 2 (dx^3) (dx^4)$
5. Show that the magnitude of a vector is invariant.
6. (i) P.T. Christoffel symbol of 1st kind is symmetric in 1st two indices.
   (ii) P.T. Christoffel symbol of 2nd kind is symmetric in 2nd two indices.
7. P.T. $[ij,k] + [kj,i] = \frac{\partial gik}{\partial xj}$
8. P.T. $[ij,h] = g_{kh} \{^k{}_{ij}\}$
9. P.T. $\frac{\partial gij}{\partial xk} = g_{lj} \{^l{}_{ik}\} + g_{li} \{^l{}_{jk}\}$
10. P.T. $\frac{\partial gim}{\partial xl} = -g^{ij} \{^m{}_{lj}\} - g^{km} \{^i{}_{kl}\}$
11. P.T. $\frac{\partial gij}{\partial xk} - \frac{\partial gjk}{\partial xi} = [kj,i] - [ij,k]$
12. S.T. the christoffel symbols Vanish identically iff g_{ij} are constant
13. Derive the law of transformation of christoffel symbol of 2nd kind.
14. If $g_{ij} = 0$ for $i \neq j$ .P.T. $g^{ii} = \frac{1}{gii}$ (no E_i)

Part – B (15 marks)

1. Find out the metric tensor in lylindrical polar co-ordinate, in E_3 space.
2. (a) If $g = [gij] > 0$ then $\{\frac{i}{il}\} = \frac{\partial \log \sqrt{g}}{\partial xl}$
   (b) P.T. $\frac{\partial\, gim}{\partial xl} = -gij \left\{\begin{matrix} m \\ lj \end{matrix}\right\} - g^{km} \left\{\begin{matrix} i \\ kl \end{matrix}\right\}$
3. Calculate the christoffel symbol of first and second kind corresponding to the metric
   $ds^2 = (dx^1)^2 + (x^1)^2 (dx^2)^2 + (a^1 \sin x^2)^2 (dx^3)^2$.
4. Evaluate the christoffel symbols for Riemannian space where $gij = 0$, $i \neq j$.
5. Derive the law of transformation of christoffet symbol of first kind.
6. Derive the law of transformation of christoffel symbol of second kind.

UNIT-III SECTION-A (6 marks)

1. Derive the covariant differentiation of covariant vector of type (0,1)
2. Derive the covariant differentiation of the contravariant vector of type (1,0)
3. P.T $A^{ij}_{j} = 1/\sqrt{g\partial}/\partial x\gamma\ [A^{ij}\sqrt{g}] + A^{jp}\{i/jp\}$

   Where a^{ij} is a tensor of type (2,0).

4. If A^{ij} is a skew-symmetric tensor –P.T
   $A^{ij}{}_{j} = 1/\frac{\sqrt{g\partial}}{\partial j}\left(AIJ\sqrt{g}\right)$
5. If Ai is a co-variant vector. P.T. $[\frac{\partial Ai}{\partial xj} - \partial Aj/\partial xi]$ *is* a covariant tensor of rank
2.
6. Let Aij and Bij be any 2 co-variant tensor of rank 2 Then S.T i) (Aij+Bij),K= Aioj,k+ Bij,K
   ii) (Aij-Bij),k = Aij,k-Bij,k
7. If A^I &B^I are tqp unit contra- variant veeton S.T they are inchined to each other at a Constant angle = A^i,k BI +bi,k Ai=0

8. If f is invariant then F,ij=Fji
9. P.T. R^i_{ijk}=-R^i iklj
10. P.T. R^i_{ljk}=0
11. P.T.$^R_{lijk}$ + $R^l{}_{jki}$+ R^l_{kij}=0
12. P.T. R_{nijk}=-R_{injk}
13. P.T.R_{nijk}=R_{jkn}i
14. P.T. R^l_{ijk},h + Ri_{khi}j +$R^l_{inj,k}$ = 0
15. P.T. the scahar Curvature of an Einstein & pace is constant.
16. Define Ricci tensor and P.T. Rij is Symmetric

SECTION-B (15marks)

1. Derive the covariant differentiation of a tensor or of type (0,2).
2. Derive the covariant derivative of a tensor of type (2,0)
3. If aij is a symmetric nonbb-ringular tensore (I aijI≠ 0) of type (0,2) such that aij,k=0P.T {I/ij}=1/2 a^{lk} $[\frac{\partial aik}{\partial xi} + \frac{\partial ajk}{\partial xi} - \partial aij/\partial axk]$
4. i) For an invariant curl (grad f0=0
   ii) If curl of a covariant vector I vanishes identically then I
5. a) If f is an invariant then Fij F,ji.
   b) If Aij = aij-aj,I iel, Aij are the components of curl of a covariant vector. S.T Aij,k +Ajk,i+ Aki,j=0
6. Derivation of Riemannian Chistoffel Tensor.
7. Derive Rieman christoffel curvature os type (1,3)
8. Derive fully curvature tensor.
6. Derive the Riemann Christoffel curvature tensor of type (0,4)
7. ii) Define Einstein space and show that the scalar curvature of an Einstein space is constant.
8. If ds^2= - $(dx^1)^2$-$(dx^2)^2$-$(dx^3)^2$ +e^{-x+} $(dx^3)^3$ · prove that all the christoffel symbols do not vanish identically but the corresponding space is flat.
9. i) Find an expression for dis R^h_{ijk}
   ii) If in a Riemannian Space Vn (n>2), R_{ij} R/Z g $^h{}_{iij}$ then P.T . The Ricci Tensor vanish Identically.
10. Explain in the detail about Intrinfic differentiation.
11. P.T. the intrinsic derivatives of g_{ij},g_{ij}, g_{ij} Vanishes identically.

UNIT – IV SPECIAL THEORY OF RELATIVITY

SECTION – A (6 MARKS)

1. Explain Galilean transformations and its characteristics.
2. State and prove Newtonian principle of relativity.
3. State and prove Maxwell’s Equation
4. Explain the ether Theory
5. Characteristic of lorentz transformation equation
6. Explain the time dilation
7. Explain longitudinal contraction with example.
8. Consider the rigid rod which has an orientation parallel to the x-axis of the inertial frame I and translates with a velocity V in the x-direction relative to this frame.
9. Explain light cone

10. A rigid rod of rest length l_0 makes an angle o^1 with the x^1 axis and is fixed in I^1 as its translates with a constant velocity V relative to I find the length of the rod and the angle between the rod on the x-axis as viewed by an observer in the inertial frame I.
11. A particle moves relative to the frame I with a velocity V^1 in a direction given by the angle $\emptyset^1$ measured from the positive x^1 axis figure given below. Find the amplitude and direction of the velocity of this particle relative to I frame

SECTION – B (15 Marks)

1. Derive the Lorentz's transformation equations.
2. derive the relativistic kinematics
3. Explain the Fin stein clock paradox.
4. Derive the addition of velocities.
5. Explain the relativistic of Doppler Effect.

UNIT – V RELATIVISTIC DYNAMICS

SECTION – A (6 MARKS)

1. Derive the Cartesian components of the relativistic momentum.
2. Derive the momentum- Energy Four- Vector.
3. Show that the R-acceleration is paralled to the Force.
4. Derive the egn of conservation of energy.
5. Derive the Lagrangian and Hamiltonian formulations.
6. Explain the momentary rest frame.
7. Suppose the round trip made by rocket from the earth to a near by star,$\propto$ centuary which is about 4 light years distance that rocket is capable of a constant acceleration g=9.50m/sec^2
   (1 light year / yr ^{2}) relative to its momentary rest frame what is the time require for the trip.
8. Derive the Energy and kinetic energy.

SECTION – B (15 Marks)

1. Derive the explicit form of f(v)?
2. Explain the Hamiltonian function.
3. Explain Rocket with contend acceleration
4. Explain rocket with constand Thrust.