D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1.

ELECTIVE - GRAPH THEORY

UNIT-I SECTION-A 2 MARKS

1. Define Graph.
2. Define Bi-Partite graph with example.
3. Define Degree of a vertex.
4. Define isomorphism.
5. Define Addition of an edge.

SECTION-B 5 MARKS

1. Prove that the sum of the degree of vertex of graph G is equal to twice the number of edges.
2. Every cubic graph has even number of vertices.
3. Show that in any graph of two (or) more people, there are always two people with exactly same number of friends in the group.
4. Let f be an isomorphic of the graph $G^{1}=(v_1,x_1)$ to the graph $G_2=(V_2,X_2)$. Let $v \epsilon V,$ then deg(v)=degf(v).
5. Prove that any graph G(p,q), $\delta \leq \frac{2q}{p} \leq \Delta$.

SECTION-C 10 MARKS

1. a) Let G be a K regular bipartite graph with bipartite (v_1,v_2) and k>0 prove that $|v_1|=|v_2|$.

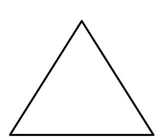
   b) Prove that self complementary graph has 4n (or) 4n+1 vertices.

2. The maximum number of edges in a (p,q) graph with no triangles is $\left[\frac{p^2}{4}\right]$.

UNIT-II SECTION-A 2 MARKS

1. Define union of two graphs.
2. Define G+V.
3. Define joining of two graphs.
4. Define composition of two graphs.
5. Given that

G_1 :



And G_2 :

Find (i) G_1+G_2 (ii) $G_1\times G_2$.

6. Define graphical sequence.
7. Define Trail, Path, Cycle.

SECTION-B 5 MARKS

1. Draw all the graphs of order 4.
2. Show that the partition p=(7,9,5,4,3,2) is not graphical.
3. Find the realisation of S, 5,5.3,3,2,2.
4. Prove that a closed walk of odd length contains a cycle.

SECTION-C 10 MARKS

1. State and Prove that Havel-Hakimi.
2. Let G_1 be a (p_1,q_1) graph and G_2 be a (p_2,q_2) graph then following results are true. Then prove that
   (i) $G_1\cup G_2$ is a (p_1+p_2 , q_1+q_2) graph.
   (ii) $G_1[G_2]$ is a $(p_1p_2 , p_1q_2+p_2^2+q_1)$.

UNIT-III SECTION-A 2 MARKS

1. Define cut vertex.
2. Define Edge cut.
3. Define Bridge.
4. Define vertex connectivity.
5. Define Block.
6. Define distance between two vertices.

SECTION-B 5 MARKS

1. Prove that a graph G of order p with $\delta(G)\geq\frac{p-1}{2}$.
2. If G is not a connected graph. Prove that G is connected.
3. Prove that for any graph G, $k(G)\leq\lambda\leq\delta(G)$.
4. The graph G is connected iff v is partioned into two different subsets v_1 and $v_2$such that there is an edge of G which has one end in v_1 and others end in v_2.

SECTION-C 10 MARKS

1. Prove that the graph G with atleast two vertices is bipartite iff all its cycle are of even length.
2. State and Prove that Whitney Theorem.

UNIT-IV SECTION-A 2 MARKS

1. Define Euler Trail.
2. Define Hamiltonions Path.
3. Define acyclic.
4. Define Forest.
5. Define Spanning Tree.

SECTION-B 5 MARKS

1. Every connected graph has a spanning tree.
2. Every tree as a centre consisting of either one or two adjacent vertices.
3. P.T Everyhamiltonion graph is 2-connected.
4. Let x and y be any 2 non-adjacent vertices of G such that degx +degy ≥ p. Then P.TG+xy is hamiltonioniff G is Hamiltonian.
5. A graph G is hamiltonioniff C(G) is Hamiltonian.

SECTION-C 10 MARKS

1. P.T C(G) is well defined.
2. State and prove that Dirac's theorem.
3. State and prove that Halls condition.

UNIT – V SECTION-A 2 MARKS

1. Define plane graph.
2. Define face.
3. Define Triangulate graph.
4. Define Girth of a graph.
5. Define colouring.
6. Define Chromatic number.
7. Define Independent set.

SECTION-B 5 MARKS

1. P.T: A graph G can be embedded on the surface sphere iff G can be embedded in the plane.
2. If G is a connected planer graph (p, q) without triangular and with p≥3 then P.T. q ≤ 2p – 4
3. P.T thekuratowski's group $K_{3,3}$ is non-planer.
4. P.T the Peterson graph is non-planer.

10 marks

1. State and prove Euler's theorem on planer graph.

2. Let G = (V, E) be simple, finite, connected undirected graph.
   Let v ϵ v(G) then the following are equivalent.
   (i) v be a cut vertex of G.
   (ii) there exists a partition of v – { v } into 2 subsets v and w $\ni$: for every u ϵ V , w ϵ W vertex v is every u – w path in G.