D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

DEPARTMENT OF MATHEMATICS

REAL ANALYSIS II

I M.Sc. MATHEMATICS EVEN SEMESTER-II

UNIT I SECTION-A 6 Marks

1. (a)assume that $f \in U(I)\ and\ g \in U(I)$
   Then$i)(f+g) \in u(I)\ and\ \int_I (f+g) = \int_I f + \int_I g$
   $ii)cf \in U(I) for\ every\ constant\ c \geq 0\ \int_I cf = c\int_I f\ \ if\ f(x) \leq g(x) a.e\ on\ I\ then\ \int_I f \leq \int_I g.$
2. State and prove levi theorem for upper function.
3. State and prove levi theorem for series of lebesgue integrable function.

SECTION-B 15 Marks

1. State and prove lebesgue dominated convergent theorem.

UNIT II SECTION-A 6 marks

1. .If $f \in M(I)\ and\ if\ |f(x)| \leq g(x) a.e\ on\ I$ for some non-negative $g \in L(I)\ then\ f \in L(I).$
2. .Let f be defined on I.Assume that $\{f_n\}$ is sequence of measurable functions on I.Show that $f_n(x) \to f(x) a.e\ on\ I\ \ then\ f\ is\ measurable\ on\ I.$

SECTION-B 15 marks

1. State and prove differentiation under the integral sign theorem.
2. i)If A and B are disjoint measurable sets then $\mu(AUB) = \mu(A) + \mu(B)$

   ii) If $\{A_1, A_2, \dots \dots . A_n\}$ is countable disjoint collection of measurable sets then
   $\mu(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mu(A_i)$

UNIT III **SECTION-A** **6 marks**

1. State and prove the Riesz fisher theorem.
2. State and prove chain rule.
3. State and prove mean value theorem.
4. State and prove weierstrass approximation theorem.
5. State and prove Fejer theorem.

SECTION-B **15 marks**

1. .State and prove taylor's theorem.
2. State and prove Reimann lebegue lemma.
3. State and prove the matrix of a linear function.

UNIT IV **SECTION-A** **6 Marks**

1. Stat and prove Dirichlet's kernel theorem.
2. If $f \in (-\infty, \infty)$ we have

$lim_{n\to\infty} \int_{-\infty}^{\infty} f(t) \frac{(1-cos\alpha t\,)}{t} dt = \int_0^{\infty} \frac{f(t)-f(-t)}{t} dt$ whenever thr lebesgue integral on the right side.

3. State and prove a sufficient condition for differentiablity
4. State and prove mean value theorem.
5. State and [rove chain rule theorem.

SECTION-B **15 Marks**

1. State and prove Jordan theorem.
2. State and prove Riemann's localization theorem
3. State and prove the matrix of a linear function.

UNIT V **SECTION-A** **6 Marks**

1. State and prove function with non-zero jacobian determinant.
2. State and prove implicit function theorem.

SECTION-B **15 Marks**

1. State and prove inverse function theorem.
2. State and prove second derivative test for extreme theorem.