D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS),VELLORE-1.

GRAPH THEORY

CLASS : I M.sc MATHS **SUBJECT CODE:15CPMA1E**

UNIT-1 SECTION-A 6 MARKS

1. Define complete graph, Adjacent edges, Cubic graph , Subgraph.

2. State and Prove Fundamental Theorem.

3. Prove that the degree of each vector of K_P is P-1.

4. For any graph prove that, $\delta \leq \frac{2E}{v} \leq \Delta$ (or) $\delta \leq \frac{2q}{p} \leq \Delta$.

5. For any simple bi-partile graph G, prove that $q(G) \leq \frac{(p(G))^2}{4}$.

6. Show that the 2 graphs are not isomorphic with example.

7. Prove that let G be a k-regular bi-graph with bi-partition (x,y) & K>0, prove that IxI=IyI .

8. If G is a (p,q) graph all of where vertices have degree K(or)K+1,If G has t vertices of degree K, Show that t=p(k+1)-2q.

9. Prove that A graph is a tree iff there is a unique path connecting a every pair of vertices.

10. Show that Any non trivial tree has atleast 2 pendent vertices.

11. prove that An edge eεG is a cut edge of G iff e contained no cycle of G.

12. Prove that A connected graph G is a tree if every edge is a cut edge.

13. To prove that every connected graph G contained a spanning tree.

14. Every tree has a centre either one vertex (or) 2 adjacent vertices.

15. Prove that if a vertex V of a tree G is a cut vertex of G<=>degree of V greater than 1.

16. Every non –trivial loopless connected graph has atleast 2 vertices are not cutvertex.

17. Prove that let T be a spanning tree of a connected graph G & Let e be an edge of G .

SECTION-B 15 MARKS

1. Prove that a graph is bi-partite iff it contains no odd cycle.

2. Prove that A connected graph of P vertices is a tree iff it has (p-1) edges.

3. State and prove characterization theorem of a tree.

UNIT-II SECTION-A 6 MARKS

1. State and prove whigney's Inequality.

2. prove that let G be a complete graph of order P then $K(G)=P-1$ and $\delta(G)=P-1$.

3. Prove that A non trivial connected graph has an eulerian trail iff it has atmost two vertices of odd degree .

4. Prove that the simple connected graph G is eulerian iff its the union of edge disjoint cycle.

5. Define Eulerian graph and Hamiltonian graph .

6. Prove that let G be a simple graph and let u&v be non-adjacent vertices of G then $d(u)+d(v)\geq v$,G is Hamiltonian <=> G+uv is Hamiltonian.

7. State and prove necessary condition for Hamiltonian graph .

8. Prove that C(G) is well defined.

9. Prove that A simple graph G is Hamiltonian iff its closure C(G) is Hamiltonian.

10. If G is simple graph of order P and $\delta(G)\geq P-2$ then prove that $K(G)=\delta(G)$.

SECTION-B 15 MARKS

1. State and prove whigney's theorem on 2 connected graph.
2. State and prove analog theorem.
3. State and prove menker's theorem.
4. Prove that A non empty connected graph is Eulerian iff it has no vertices of odd degree.
5. State and prove characterization theorem for Eulerian .
6. State and prove Dirac theorem.

UNIT-III SECTION-A 6 MARKS

1. Define matching, maximal matching, alternating path augmenting path.

2. State and prove Berge's Theorem .

3. Prove that let G is a K-regular bi-Partile graph with $k>0$ then G has a perfect matching.

4. Prove that let M be a matching and K be a covering $|M|\leq|K|$. Then M is a Maximum matching and k is a minimum theorem .

5. State and prove Konig's minimum theorem.

6. Let G be a connected graph (i.e) not an odd cycle then G has 2-edge colouring in which both colours are represented at each vertex of degree atleast two .

7. Prove that if G is bi-partite then $\Psi'=\Delta$.

SECTION-B 15 MARKS

8. State and Prove Hall's Theorem.

9. State and prove Vizing's Theorem.

10. Prove that Let $Ç=(E_1,E_2.....E_K)$ be an optimal k-edge colouring of G if there is a vertex u in G and colours i and j such that i is not represented at u and j is represented atleast twice at u, then the component of G $[E_i U E_j]$ that contains u is an odd cycle.

UNIT-IV SECTION-A 6 MARKS

1. Define independent sets and cliques Ramsey's number.

2. Prove that for any graph G or γ vertices $\alpha+\beta=\gamma$.

3. Prove that for any two positive integers m,n $r(k,l)\leq\begin{bmatrix} k+l-2 \\ l-1 \end{bmatrix}$.

4. Prove that for any integer $k\geq2$, $r(k,k)\geq2^{k/2}$.

5. Prove that if G is k-critical then $\delta\geq k-1$.

6. If G is simple then $\prod_k(G)=\prod_k(G-e)-\prod_k(G.e)$ for any edge of G .

7. Prove that for any graph G, $\prod_k(G)$ is a polynomial in k of degree v with integer co-efficients leading term k^v and constant term zero further more ,the coefficient of $\prod_k(G)$ alternative in sign.

SECTION-B 15 MARKS

1. State and prove Erdas Szekeros theorem .
2. State and prove Ramsey's number.
3. State and prove Gall's theorem.
4. State and prove Brooks theorem.

UNIT-V SECTION-A 6 MARKS

1. Prove that K_5 is non planar graph .
2. Prove that $K_{3,3}$ is non-planar graph.
3. State and prove Euler's formula for planar graph.
4. State and prove necessary and sufficient condition for planar graph.
5. State and prove Kuratowshi graph of K_5 & $K_{3,3}$ are non-planar.
6. Prove that in a simple planar graph $\delta \leq 5$.

SECTION-B 15 MARKS

1. State and Prove color theorem.
2. State and prove headwood's five theorem.
3. Prove that every planar graph is 5 vertex colourable.
4. i) Prove that a planar graph is maximum iff all its faces are triangle.
5. ii) Prove that every planar graph is 4 vertex colourable.