D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1.

SOLID GEOMETRY

PLANE - UNIT I (2 marks)

1. Define Plane?

2. Find the direction cosines of the normal to the plane (a) $2x-3y+6z=7$ (b) $x+2y+2z-1=0$

3. Find the equation of a plane which passes through the point $(2,3,4)$?

4. Find the intercept which the plane makes with a coordinate axis $4x-3y+2z-7=0$?

5. Find the angle between the planes $2x+4y-6z=11$, $3x+6y+5z+4=0$?

6. Find the equation of the plane which passes through the point $(2,-4,5)$ and parallel to the plane $4x+2y-7z+6=0$?

7. Find the equation of the plane in which $(12,-4,-3)$ are the coordinates of the foot of the perpendicular?

8. Find the equation of the plane passing through $(3,-3,1)$ and its normal to the line joining points $(3,2,-1)$ and $(2,-1,5)$?

9. Find the equation of the plane through the origin and the line of intersection of the planes $3x-y+2z=4$, $x+y+z=1$?

10. Find the length of the perpendicular drawn from the point $(0,1,-1)$ to the plane $2x-y+2z-3=0$?

11. Find he distance between parallel planes $12x+3y-4z+15=0$, and $12x+3y-4z+2=0$?

SECTION-B   5 MARKS

1. Find the equation of the plane passing through the points $P(2,2,-1)$, $Q(3,4,2)$, $R(7,0,6)$?

2. Find equation of the plane passing through the points $(2,2,1)$ and $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$?

3. Find the equation of the plane passing through the points $(-1,1,1)$ $(-1,3,2)$ and perpendicular to the $x+2y+2z=5$?

4. Find the equation of the plane $(1,1,2)$ and perpendicular to the planes $2x-2y-4z=3$, and $3x+y+6z=4$?

5. Show that the given points are coplanar $(0,2,-4)$ $(-1,1,-2)$, $(-2,3,3)$, and $(-3,-2,1)$?

6. Plane makes the coordinate axes A,B,C such that the centroid of the triangle ABC is $(1,\frac{1}{2},\frac{1}{2})$ find the equation of the plane?

7. A plane meets the coordinate axis at A,B,C. The centriod of the triangle is α,β,r . Find the equation of the plane?

8. A variable plane passes through a fixed point a,b,c and meets the coordinate axis in A,B,C Show that the locus of the point common on the plane through A,B,C parallel to the coordinate plane is $a/x+b/y+c/z=1$?

9. Find the equation of the plane through the line of intersection of the two planes $x+y+z-1=0$ and $2x+y-3y+2=0$ and through the point $(1,1,1)$?

10. Find the equation of the plane passing through the line of intersection of the planes $2x+y+3z-4=0$, and $4x-y+5z-7=0$ and is perpendicular to the plane $x+3y-4z+6=0$?

11. Find bisector plane between $3x-6y+2z+5=0$ and $4x-12y+3z-3=0$?

SECTION-C   10 MARKS

1. Find the equation of the plane making intercepts a,b,c and axis ox,oy,oz.

2. To find equation of the plane in term of p, the length of the perpendicular from the origin to it and l,m,n are the direction cosines of the perpendicular?

3. Find the equation of the plane passing through the points (2,2,1), (1,-2,3) and also parallel to the line joining points (2,1,-3) and (-1,5,-8)?

4. Find the equation of the plane passing through the points (2,2,1), and (1,-2,3) and also parallel to x-axis?

5. A variable plane which remains at a constant distance 3p from the origin and cut the coordinate axis at A,B,C . Show that the centroid of the triangle is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$?

6. Find the ratio in which the plane ax+by+cz+d=0 divides the line joining the points x_1,

7. Find the equation of plane passing through the line of intersection of two planes 2x-5y+z=3 and x+y+4z-5=0 and parallel to the plane x+3y+6z=0

8. Find the equation of plane passing through the lines of intersection of the two planes x+2y+3z+2=0,and 2x+3y-z+3=0 and parallel to the lines whose direction ratios are (1,1,1)?

9. Show that the plane 11x+19y+31z-18=0 bisects the acute angle between the planes x+2y+2z-3=0 and 3x+4y+12+1=0?

10. Obtain the equation of planes which bisects the angles between the planes 2x+2y-z+1=0 and 3x+4y+12z-2=0 and check which plane bisects the acute angle in two planes?

11. Show that origin lies in the acute angle between the planes   x+2y+2z=9, and 4x-3y+12z+13=0 find the planes bisecting the angle between them and point out which plane bisects the obtuse angle?

UNIT-II   STRAIGHT LINE   2 Marks

1. Define straight line?

2. Write down formula for straight line in symmetric form?

3. Find the equation of the line joining the points (1,-1,2) and (4,2,3)?

4. Find the Angle between the lines $\dfrac{x+1}{2} = \dfrac{y+3}{2} = \dfrac{z-4}{-1}$ and $\dfrac{x-4}{1} = \dfrac{y+4}{2} = \dfrac{z+1}{2}$

5. Find the equation of the line passing through the point (3,2,-8) and perpendicular to the plane -3x+y+2z-2=0?

6. Write down formula for angle between a line and a plane?

7. Find the angle between the line $\dfrac{x+1}{2} = \dfrac{y-2}{3} = \dfrac{z-1}{6}$ and the plane 3x+y= $\dfrac{x}{2} = \dfrac{y}{1} = \dfrac{z}{3}$

8. state the condition that two straight lines are coplanar.

SECTION-B   5 MARKS

1. Find the symmetrical form of the equation of line x+y+z-1=0=2x-y+3z+1?

2. Show that the line $\dfrac{x}{2}=\dfrac{y}{1}=\dfrac{z}{3}$ and 3x+2y+z-5=0=x+y-2z-3 are perpendicular?

3. Find the length of the perpendicular P(5,4,-1) and to the line $\dfrac{x-1}{2}=\dfrac{y}{9}=\dfrac{z}{5}$?

4. Find a point in between the line joining points (11,0-1) and (-9,4,5) is met by the perpendicular drawn through the origin?

5. Find perpendicular distance from A(-1,3,9) to the line $\dfrac{x-13}{5}=\dfrac{y+8}{-8}=\dfrac{z-31}{1}$?

6. Find the point where the line $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ meets the plane x+y+z=15?

7. Find the direction cosine of the straight line $\dfrac{2x-1}{7}=\dfrac{3y+2}{6}=\dfrac{7z+1}{10}$?

8. Find the equation of line which can be drawn from the point (2,-1,3) to intersect lines $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and $\dfrac{x-4}{4}=\dfrac{y}{5}=\dfrac{z+3}{3}$ at right angles?

9. Find the equation of the projection of the line 3x+4y+3=0=x+2y+2z-1 on the plane x+2y+3z+1=0?

10. Find the distance of the point (1,2,-4) from the line $\dfrac{x-3}{2}=\dfrac{y-1}{-5}=\dfrac{z+2}{3}$?

11. Find the equation of the line with direction ratios (6,-3,2) and passing through the point (2,-1,-1). What are the co-ordinates of the two points on this line distance 7 units from the given points?

SECTION-C   10 MARKS

1. Show that the lines $\dfrac{x+1}{7}=\dfrac{y-4}{1}=\dfrac{z+3}{-4}$ and $\dfrac{x+10}{8}=\dfrac{y-1}{2}=\dfrac{z+1}{-3}$ are coplanar and find the equation of plane containing them?

2. Show that the following lines are coplanar and find the point of intersection and equation of plane of coplanarity by 3x-y-z+2=0=x-2y+3z-6 and 3x-4y+3z-4=0=2x-2y+z-1?

3. Find the shortest distance between the two lines $\dfrac{x-3}{-1}=\dfrac{y-4}{2}=\dfrac{z+2}{1}$ and $\dfrac{x-1}{1}=\dfrac{y+7}{3}=\dfrac{z+2}{2}$. also find the equation of the shortest distance?

4. Find the point where the line x-2y+z=1, x+2y-2z=5 intersects the plane 2x+2y+z+6=0.

5. Find the equation of plane passing through the line of intersection of 2x+y-z=3 and 5x-3y+4z+9=0 and parallel to the line $\dfrac{x-1}{2}=\dfrac{y-3}{4}=\dfrac{z-5}{5}$?

6. Show that the lines $\dfrac{x+4}{3}=\dfrac{y+6}{5}=\dfrac{z-1}{-2}$ and 3x-2y+z+5=0=2x+3y+4z-4 are coplanar. Find their point of intersection and the plane in which they lie?

7. Find the equation of the line which is the projection of the line $\dfrac{x-2}{0}=\dfrac{y+3}{-2}=\dfrac{z-3}{1}$ on the plane x-2y-z+1=0.

8. Show that the shortest distance between the lines $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and $\dfrac{x-2}{3}=\dfrac{y-4}{4}=\dfrac{z-5}{5}$

is $\dfrac{1}{\sqrt{6}}$?

9. Find the feet get the common perpendicular drawn to the following lines and find the length of

the common perpendicular $\dfrac{x+2}{2}=\dfrac{y+6}{3}=\dfrac{z-34}{-10}$ and $\dfrac{x+6}{4}=\dfrac{y-7}{-3}=\dfrac{z-7}{-2}$?

10. Find the shortest distance between the lines $\dfrac{x-5}{3}=\dfrac{y-6}{-4}=\dfrac{z-9}{1}$ and 2x-2y+z-3=0=2x-y+2z-9?

11. Find the shortest distance between the lines 3x-9y+5z=0=x+y-z, 6x+8y+3z-10=0=x+2y=z-3?

SPHERE – Unit III   SECTION-A   2 Marks

1. Define sphere?

2. To find the equation of the sphere whose center is (a,b,c)and radius r.

3. Find the equation of the sphere whose center is (-6,1,3) and radius 4.

4. Find the equation of the sphere on the line joining the points (2,7,5) and (8,-5,1) as diameter.

5. To find its radius and center. $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ this equation of sphere?

6. Define coaxial system of sphere

7. Write the condition for two spheres cut orthogonally.

8.Find the equation of the tangent plane at (1,2,3) to the sphere x²+y²+z²+x+y+z=20

9.Find the equation of sphere through(a,0,0),(0,b,0),(0,0,c) and (0,0,0)

SECTION-B   5 MARKS

1. Find the co-ordinates of the center and radius of sphere $2x^2+2y^2+2z^2-2x+4y+2z+3=0$

2. Find the equation of the sphere which as its center (6,-1,2) and it touches the plane 2x-y+2z-2=0?

3. If $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ this equation always represent a sphere and to find its radius and center?

4.Find the equation of the sphere through (0,0,0),(a,0,0), (o,b,0), (0,0,c)

Find the co-ordinates of central of the circle $x^2+y^2+z^2=30, x+2y+3z=14$?

4. Find the equation of the sphere which passes through the point (1,-2,3) and through the circle z=0, $x^2 + y^2 + z^2 = 9$?

5. Find the equation of the sphere through out a circle $x^2 + y^2 + z^2 + 4x - 2y + 4z - 10 = 0$, $x^2 + y^2 + z^2 - 4 = 0$ and through out the point (2,1,1)?

6. Obtain the radius axis for the sphere $(x-2)^2 + y^2 + z^2 = 1$, $x^2 + (y-3)^2 + z^2 = 0$ and $(x+2)^2 + (y+1)^2 + (z-2)^2 = 6$?

7. Obtain the equation of sphere passes through (1,0,0), (0,1,0), (0,0,1) and has its center on the plane x+y+z=6?

8. Prove that the equation of the sphere described on the line joining the points (2,-1,4) and (-2,2,-2) as diameter is $x^2 + y^2 + z^2 - y - 2z - 14 = 0$?

SECTION-C   10 MARKS

1. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$, $2x - y + 2z = 5$ for a great circle?

2. Find the limiting point of the axial system of spheres determined by the sphere $x^2 + y^2 + z^2 + 3x - 3y + 6 = 0$, $x^2 + y^2 + z^2 - 6y - 6z + 6 = 0$?

3. Find the equation of the sphere through the origin and coaxial with the sphere $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 + x + 2y + 3z - 5 = 0$?

4. Find the equation of the sphere having radius 3 and passing through circle $x^2 + y^2 + z^2 + 2x + 2z - 9 = 0$, $2x + 2y + 2z - 3 = 0$

5. Show that the intersection of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0$ and plane x+2y+2z-20=0 is the circle of the radius $\sqrt{7}$ with the center at the point (2,4,5)?

6. Find the equation of the sphere passes through $x^2 + y^2 + z^2 - 2x - 4y = 0$, x+2y+3z=8 and touches the plane 4x+3y=25?

7. Find the equation of sphere passes through (-4,0,-2), (-1,2,-1) and having the center on the line 5z+2x=0=2y-3z?

8. Find the equation of the sphere which has its center on the plane 5x+y-4z+3=0 and passing through the circle $x^2 + y^2 + z^2 - 3x + 4y - 2z + 8 = 0$ and 4x-5y+3z-3=0?

CONE – Unit IV   SECTION-A   2 Marks

1. Define cone?

2. Define Quadric cone ?

3. Define Right circular cone?

4. To prove that the equation of a Quadric cone whose vertex is at the origin is homogeneous.

5. To show that the equation of a quadric cone passing through the x,y,z axes is of the form fyz+gzx+hxy=0?

6. Prove that the equation of the cone whose vertex is the origin and base curve z=k, f(x,y)=0 is

$$f\left(\frac{xk}{z},\frac{yk}{z}\right)=0?$$

7. Equation of a right circular cone with its vertex (α,β,γ) its axis is at the line $\dfrac{x-\alpha}{l}=\dfrac{y-\beta}{m}=\dfrac{z-\gamma}{n}$

and semi-vericle angle is θ and its value is equal to $\left[l(x-\alpha)+m(y-\beta)+n(z-\gamma)\right]^2=$

$\left[(l^2+m^2+n^2)(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2\right]\cos^2\theta?$

8. Find the equation of the cone whose vertex is the origin and the guiding curve is

$$\frac{x^2}{4}+\frac{y^2}{9}+z^2=1, x+y+z=1?$$

9. Define enveloping cone?

10. To obtain the equation of the cone whose vertex is at A(x,y,z) and which envelops the sphere?

11. Define Reciprocal cone?

12. Find the enveloping cone of the sphere $x^2+y^2+z^2-2x+4z=1$ with its vertex (1,1,1)?

13. The condition that the plane $lx+my+nz=P$ is a tangent plane to the coincoid

$ax^2+by^2+cz^2=1?$

SECTION-B   5 MARKS

1. Find the equation of the cone whose vertex is at the point (α,β,γ) and whose generators intersects

the conic $x^2+y^2+2gx+2fy+c=0, z=0?$

2. Find the equation of the cone whose vertex is at the point (1,1,0) and whose guiding curve is

$x^2+z^2=4, y=0?$

3. Find the equation of the cone with the vertex is origin and passing through $x^2+y^2=4, z=2?$

4. Show that the equation of a right circular cone whose vertex is 0 axis oz and semi-vertical angle

α is $x^2+y^2=z^2\tan^2\alpha?$

5. Find the equation of cone whose vertex 0 and base curve, the conic in which the surface

$ax^2+by^2+cz^2=1$ is cut by the plane $l_1x+m_1y+n_1z=p?$

6. Fine the equation of the right circular cone whose vertex is at origin and axis is $\dfrac{x}{1}=\dfrac{y}{2}=\dfrac{z}{3}$ and

semi-verticle angle is $30°?$

7. Find the equation of the Right circular cone having the line x=y=z as it axis and 2x=-5y=3z the line as a generator?

8. Find the equation of a right circular cone whose vertex is (1,2,3) axis is the line x-1=y-2=z-3 and generator has directions (1,0,-2)?

9. Find the equation of the cone whose vertex is the origin and the base is the circle

$y^2+z^2=b^2, x=a$ further show that the section of the cone by a plane parallel to z=c, plane is a hyperbola?

10. Find the equation of the tangent plane which passes through the line x+9y-3z=0=3x-3y+6z-5, and touch the conicoid $2x^2 - 6y^2 + 3z^2 = 5$?

SECTION-C   10 MARKS

1. Find the equation of the cone with vertex at (1,1,1) and passing through the curve of intersection of $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$?

2. Find the equation of a cone whose vertex is the point (α,β,γ) and whose generating lines pass through the conic $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, z = 0$. If the section of this cone by the pane x=0 is a rectangular hyperbola, show that the locus of p is $\dfrac{x^2}{a^2} + \dfrac{y^2 + z^2}{b^2} = 1$.

3. The plane $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ meets the axis at A,B and   C. Find the equation of the cone whose the guiding curve is the circle ABC?

4. Prove that the equation $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$ represents a cone whose vertex is (-1,-2,-3)?

5. Show that $x^2 - 2y^2 + z^2 + 4yz + 2zx + 6xy - 12x - 10y - 10z + 21 = 0$ represents a cone find its vertex?

6. If $ax^2 + by^2 + cz^2 + 2fyz + 2gx + 2hxy = 0$ represents a right circular cone. Show that its axis is fx=gy=hz. Find the semi-vertical angle?

7. Determine the angle between the line of intersection of the plane x-3y+z=0 and the cone $x^2 - 5y^2 + z^2 = 0$?

UNIT- V   CYLINDER   SECTION-A   2 MARKS

1. Define cylinder?

2. Define Enveloping cylinder?

3. Find the equation of the right circular cylinder of radius 3 and axis $\dfrac{x-1}{2} = \dfrac{y-3}{2} = \dfrac{z-5}{-1}$

SECTION-B   5 MARKS

1. Find the equation of the cylinder whose generators are parallel to the line $\dfrac{x}{1} = \dfrac{y}{-2} = \dfrac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$?

SECTION-C   10 Marks

1.Find the equation of the cylinder whose generators are parallel to z-axis and the guiding curve is $ax^2 + by^2 = cz, lx + my + nz = P$?

2. Find the equation of cylinder whose generator are parallel to y-axis and which passes through a curve of intersection of $x^2 + y^2 + z^2 = 3 and x + y + z = 3$

3. Find the equation of the right circular cylinder whose radius 3 with axis $\dfrac{x+2}{3} = \dfrac{y-4}{6} = \dfrac{z-1}{2}$?

4. Find the equation of the right circular cylinder described on the circle through the points (a,0,0), (0,a,0), (0,0,a) as a guiding curve?

5. Find the equation of enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x - 4y = 1$ having its generators parallel to the line x=y=z?