Reg.No : | | | | | | | | | | | |

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

APRIL – 2018 **15CPMA4D**

DIFFERENCE EQUATIONS

Time : 3 Hours **Max. Marks: 75**

SECTION – A (5 x 6 = 30)

Answer ALL the questions.

1. *(a) State and prove Abel's theorem.*

(Or)

(b) Show that the operators Δ and E are linear.

2. *(a) State and prove variation of constant formula.*

(Or)

(b) Let B be a $K \times K$ non singular matrix. Let m be any positive integer then there exist some $K \times K$ matrix C such that $C^m = B$.

3. *(a) Obtain the inverse Z transformation of* $\bar{x}(z) = \dfrac{z(z-1)}{(z-2)^2(z+3)}$

(Or)

(b) The zero solution of $x(n+1) = Ax(n) + \sum_{j=0}^{n} B(n-j)x(j)$ *is uniformly stable, if and only if*

(i) $z - A - \bar{B}(z) \neq 0 \;\forall\; |z| > 1$

(ii) If z_r *is a zero of g(z) with* $|z_r| = 1$ *then the residue of* $z^n g^{-1}(z)$ *at* z_r *is bounded as* $n \to \infty$.

4. *(a) Find the Asymptotic estimates of a fundamental set of solutions of*

$$y(n+1) = [A + B(n)]y(n)$$

Where $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ $B(n) = \begin{bmatrix} \frac{1}{n^2+1} & 0 & (0.5)^n \\ 0 & (0.2)^n & 0 \\ e^{-n} & 0 & \frac{\log n}{n^2} \end{bmatrix}$.

(Or)

(b) Verify whether the difference system $x(n+1) = D(n)x(n)$ *with*

$$D(n) = \begin{bmatrix} 1+\frac{1}{n+1} & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & n+1 & 0 \\ 0 & 0 & 0 & \frac{1}{n+2} \end{bmatrix}$$ *possess an ordinary dichotomy.*

5. (a) If $c(n) \geq a(n) > 0$ for all n>0 and z(n)>0 is a solution of $c(n)z(n) + \frac{1}{z(n-1)} = 1$ then the equation

$$a(n)y(n) + \frac{1}{y(n-1)} = 1$$ has a solution $y(n) \geq z(n) > 1 \forall n \in z^{+}$.

(Or)

(b) Determine the oscillatory behaviour of all solutions of $\Delta[n\Delta x(n-1)] - \frac{1}{n}x(n) = 0$.

SECTION – B (3 x 15 = 45)

Answer any THREE of the following questions.

6. If $1+P_1+P_2>0$, $1-P_1+P_2>0$, $1-P_2>0$ are necessary and sufficient condition for the equilibrium point of the equation $y(n+2)+P_1 y(n+1)+P_2 y(n)=0$.

7. Solve the system $x(n+1) = Ax(n)$ where $x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $A = \begin{pmatrix} 3/2 & 1/2 & 1/2 \\ 1/2 & 5/2 & -1/2 \\ 0 & 1 & 2 \end{pmatrix}$

8. Solve the difference equation $x(n+4) + 9x(n+3) + 30x(n+2) + 24x(n) + 44x(n+1) = 0$, $x(0) = 0, x(1) = 0, x(2) = 1, x(3) = -10$.

9. State and prove POINCARE theorem.

10. State and prove STRUM SEPERATION theorem.

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