Reg.No : | | | | | | | | | | | |

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER - 2016 **15CAMA1A /15CAMA3A**

ALLIED : MATHEMATICS - I

Time : 3 Hours **Max. Marks : 75**

SECTION – A (10 x 2 = 20)

Answer ALL the questions.

1. *Prove that* $\frac{e^2-1}{e^2+1} = \frac{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\cdots}{1+\frac{1}{2!}+\frac{1}{4!}+\cdots}$.
2. *Find the co - efficient of* x^n *in the expansion of* $1 + \frac{(1+ax)}{1!} + \frac{(1+ax)^2}{2!} + \cdots \infty$.
3. *Solve* $2x^3 - 15x^2 + 46x - 42 = 0$, *given that* $3 - i\sqrt{5}$ *is a root.*
4. *If 1, 2, 3 are the roots of the equation* $x^3 - 6x^2 + ax - 6 = 0$. *Find the value of* a.
5. *Define symmetric matrix. Give example.*
6. *State Cayley - Hamilton theorem.*
7. *Show that* $\cos h^2 x - \sin h^2 x = 1$.
8. *Expand* $\cos n\theta$ *in powers by* $\cos\theta$ *and* $\sin\theta$.
9. *If* $xy = ae^x + be^{-x}$, *then prove that* $xy_2 + 2y_1 - xy = 0$.
10. *Write the formula for radius of curvature in terms of polar co - ordinates.*

SECTION – B (5 x 5 = 25)

Answer any FIVE of the following questions.

11. *When x is small, show that* $\sqrt{x^2+4} - \sqrt{x^2+1} = 1 - \frac{x^2}{4} + \frac{7x^4}{64}$ *nearly.*
12. *Prove that* $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} \ldots = \frac{3e}{2}$.
13. *If* α, β, γ *are the roots of the equation*

    $x^3 + px^2 + qx + r = 0$.

    Find the value of *i)* $\sum \alpha^2\beta$ *ii)* $\sum \alpha^2$ *iii)* $\sum \alpha^3$.
14. *Diminish by 3 the roots of* $x^4 + 3x^3 - 2x^2 - 4x - 3 = 0$.
15. *Find characteristic roots of the matrix* $A = \begin{pmatrix} 1 & 2 & 1 \\ -2 & 4 & 3 \\ 1 & 0 & 2 \end{pmatrix}$.
16. *Expand* $\tan 6\theta$ *in terms of* $\tan\theta$.
17. *Find the* n^{th} *derivative of* $y = \cos(ax+b)$.
18. *If* $u = x^2 - y^2$, $v = xy$. *Prove that* $\frac{\partial(u,v)}{\partial(x,y)} = 2(x^2+y^2)$.

SECTION – C (3 x 10 = 30)

Answer ALL the questions.

19. *(a) i) Prove that* $2\left[\frac{1}{2n+1}+\frac{1}{3}\frac{1}{(2n+1)^3}+\frac{1}{5}\frac{1}{(2n+1)^5}+\ \dots\right]=\log\left(\frac{n+1}{n}\right)$.

ii) Prove that $\log_2 e\ -\log_4 e\ +\log_8 e-\log_{16} e+\cdots=1$.

(Or)

(b) Solve $6x^5-x^4-43x^3+43x^2+x-6=0$.

20. *(a) Find the real positive root of* $x=\sqrt{12}$ *by Newton's method.*

(Or)

(b) Verify Cayley - Hamilton theorem and hence find the inverse for $\begin{pmatrix}1 & 0 & 3\\ 2 & 1 & -1\\ 1 & -1 & 1\end{pmatrix}$.

21. *(a) Prove that* $\sin^5\theta=\frac{1}{16}(\sin 5\theta-5\sin 3\theta+10\sin\theta)$.

(Or)

(b) If $y=\sin^{-1}x$. *Prove that* $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$.

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