D. K. M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1
SEMESTER EXAMINATIONS
NOVEMBER - 2017   15CMA1A / CMA1A

ALGEBRA

Time : 3 Hours   **Max. Marks : 75**

SECTION – A (10 x 2 = 20)
Answer ALL the questions.

1. If α and β are the roots of $2x^2 + 3x + 5 = 0$, find $\alpha + \beta$, $\alpha\beta$.

2. Multiply the roots of $x^3 - 3x + 1 = 0$ by 10.

3. Write down the formula for Newton's method.

4. State Descartes rule of signs.

5. Find the coefficient of x^n in the expansion of e^{a+bx}.

6. Show that $\left(\dfrac{1+2x}{1+x}\right)^n = 1 + n\left(\dfrac{x}{1+2x}\right) + \dfrac{n(n+1)}{2!}\left(\dfrac{x}{1+2x}\right)^2 + \cdots$

7. Prove that $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

8. Find the eigen values of $A = \begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$.

9. Find the number of integers less than 500 and prime to it.

10. Find the number of divisors of 140 and their sum.

SECTION – B (5 x 5 = 25)
Answer any FIVE of the following questions.

11. Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$ given that $2 + i\sqrt{3}$ is a root.

12. If α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$, find the value $\sum \alpha^2$, $\sum \alpha^2\beta$.

13. Evaluate $\sqrt{12}$ to four decimal place by Newton's method.

14. Sum of the series $\dfrac{5}{1!} + \dfrac{7}{3!} + \dfrac{9}{5!} + \cdots \infty$.

15. Find the sum to infinity the series $1 + \dfrac{2}{6} + \dfrac{2.5}{6.12} + \dfrac{2.5.8}{6.12.18} + \cdots \infty$.

16. Find the rank of the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & -4 \\ -3 & 1 & -2 \end{bmatrix}$.

17. Find the eigen values of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

18. Find the highest power of 2 in 1000!.

Answer ALL the questions.

19. (a) Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2 and solve the transformed equation.

(Or)

(b) Solve $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

20. (a) Find the negative root of the equation $x^3 - 9x^2 + 18 = 0$, correct to two decimal places by Horner's method.

(Or)

(b) Show that $\dfrac{1}{1.2.3} + \dfrac{1}{3.4.5} + \dfrac{1}{5.6.7} + \cdots \infty = log\, 2 - 1/2$.

21. (a) Verify Cayley Hamilton theorem for $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.

(Or)

(b) Prove that $(18!) + 1$ is divisible by 437.

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