

Reg. No. 

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**D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**  
**SEMESTER EXAMINATIONS**  
**NOVEMBER – 2016**  
**ALGEBRA**

**15CMA1A/CMA1A**

**Time : 3 Hrs**

**Max. Marks : 75**

**SECTION-A (10 x 2 = 20)**

**Answer ALL questions.**

1. Form the equation whose roots are  $1 - \sqrt{2}, 2$ .
2. Remove the fractional co-efficients from the equation  $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$ .
3. Write the Descartes' rule of signs for negative roots.
4. Find the rational root of  $2x^3 - x^2 - x - 3 = 0$  and hence complete the solution of the equation.
5. Show that  $\left(\frac{1+2x}{1+x}\right)^n = 1 + n\left(\frac{x}{1+2x}\right) + \frac{n(n+1)}{2!}\left(\frac{x}{1+2x}\right)^2 + \dots$
6. Prove that  $\frac{e+1}{e-1} = \frac{\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \dots}{\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \dots}$
7. Define symmetric matrix with an example.
8. State Cayley Hamilton theorem
9. Prove that  $2^{1000} = 1 \pmod{17}$ .
10. State Fermat theorem.

**SECTION-B (5 x 5 = 25)**

**Answer any FIVE of the following questions.**

11. Solve the equation  $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$  given that one of the roots is  $1 - \sqrt{5}$ .
12. Find the roots of the equation  $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$ .
13. Determine completely the nature of the roots of the equation  $x^5 - 6x^2 - 4x + 5 = 0$ .
14. Sum the series  $1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \dots$
15. Prove that  $\sum_{n=0}^{\infty} \frac{n^2+3n+4}{n!} = 9e$ .
16. Test whether the following system of equations are consistent or not
$$\begin{aligned}x - y + z &= -9; \\2x - y + z &= 4; \\3x - y + z &= 6; \\4x - y + 2z &= 7.\end{aligned}$$
17. Find the eigen values and eigen vectors of the matrix  $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ .
18. Show that  $\lfloor 18 \rfloor + 1$  is divisible by 435.

**SECTION-C (3 x 10 = 30)**

**Answer ALL questions.**

19. (a) (i) Solve  $x^3 - 12x^2 + 39x - 28 = 0$  whose roots are in A.P.

(ii) Solve the equation  $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$ .

(Or)

(b) Find the real positive root of the equation  $x^3 + 24x - 50 = 0$  correct to 3 places of decimals by Horner's Method.

20. (a) Find the sum to infinity of the series  $\frac{1}{9.18} + \frac{1.3}{9.18.27} + \frac{1.3.5}{9.18.27.36} - \dots$

(Or)

(b) Verify Cayley Hamilton theorem for  $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$  and find its inverse.

21. (a) Diagonalise the matrix  $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ .

(Or)

(b) (a) Prove that  $a^{12} - b^{12} \equiv 0 \pmod{1365}$ .

(b) Show that  $8^{\text{th}}$  power of any number is of the form  $17m$  or  $17m \pm 1$ .

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