

TRIGONOMETRY

Time : 3 Hours

Max. Marks : 75

SECTION – A (10 x 2 = 20)

Answer ALL the questions.

1. Expand $\sin 4\theta$ in powers of $\cos\theta$ and $\sin\theta$.
2. Expand $\tan 4\theta$ in terms of $\tan\theta$.
3. Write down the expansion of $\sin\theta$ in terms of θ .
4. Find $\lim_{x \rightarrow 0} \frac{\sin 2x - 2\sin x}{x^3}$.
5. Prove : $\operatorname{sech}^2 x = 1 - \tanh^2 x$.
6. Prove : $\sinh^{-1} x = \log(x + \sqrt{x^2 + 1})$.
7. Find $\operatorname{Log}(1 + i)$.
8. Show that $i^n = e^{-(4n+1)\pi/2}$ where n is an integer.
9. Sum to infinity the series : $1 + \frac{c}{1!} \cos\alpha + \frac{c^2}{2!} \cos 2\alpha + \frac{c^3}{3!} \cos 3\alpha + \dots$.
10. Write down the Gregory's series.

SECTION – B (5 x 5 = 25)

Answer any FIVE of the following questions.

11. Expand $\cos 6\theta$ in terms of $\sin\theta$.
12. Show that $2^8 \sin^9 \theta = \sin 9\theta - 9\sin 7\theta + 36\sin 5\theta - 84\sin 3\theta + 126\sin \theta$.
13. Prove : $-2^4 \cos^3 \theta \sin^2 \theta = \cos 5\theta + 3\cos 3\theta - 2\cos \theta$.
14. Separate into real and imaginary parts : $\tan^{-1}(x + iy)$.
15. If $\tan \frac{x}{2} = \tanh \frac{x}{2}$, show that $\cos x \cosh x = 1$.
16. Find the value of $\operatorname{Log} \left(\frac{1 + \cos\theta + i\sin\theta}{\cos\theta - 1 + i\sin\theta} \right)$.
17. Sum to infinity the series $1 + \frac{c^2}{2!} \cos 2\alpha + \frac{c^4}{4!} \cos 4\alpha + \frac{c^6}{6!} \cos 6\alpha + \dots$
18. Find the sum to n terms of the series: $\tan^{-1} \frac{1}{1+1^2} + \tan^{-1} \frac{1}{1+2^2} + \tan^{-1} \frac{1}{1+3^2} + \dots$.

SECTION – C (3 x 10 = 30)

Answer ALL the questions.

19. (a) Prove : $\cos 8\theta = 1 - 32\sin^2 \theta + 160\sin^4 \theta - 256\sin^6 \theta + 128\sin^8 \theta$.

(Or)

(b) Prove : $2^{11} \cos^7 \theta \sin^5 \theta = \sin 12\theta + 2\sin 10\theta - 4\sin 8\theta - 10\sin 6\theta + 5\sin 4\theta + 20\sin 2\theta$.

20. (a) i) If $\cos(\theta + i\phi) = k(\cos \alpha + i \sin \alpha)$, prove that $\cos 2\theta + \cosh 2\phi = 2k^2$.

ii) If $\frac{\sin \theta}{\theta} = \frac{5045}{5046}$, show that θ is equal to $1^{\circ}58'$ nearly.

(Or)

(b) If $\tan(\theta + i\phi) = \cos \alpha + i \sin \alpha$, show that

$$(i) \theta = \frac{n\pi}{2} + \frac{\pi}{4} \quad (ii) \phi = \frac{1}{2} \log \tan \left[\frac{\pi}{4} + \frac{\alpha}{2} \right].$$

21. (a) If $\log \sin(\theta + i\phi) = A + iB$, show that i) $2e^{2A} = \cosh 2\phi - \cos 2\theta$

$$ii) \cos(\theta - B) = e^{2\phi} \cos(\theta + B).$$

(Or)

(b) Prove that $1 - \frac{1}{2} \cos \theta + \frac{1.3}{2.4} \cos 2\theta - \frac{1.3.5}{2.4.6} \cos 3\theta + \dots \dots \infty = \frac{\cos \theta/4}{\sqrt{2 \cos \theta/2}}$.

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