

**D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE – 1****SEMESTER EXAMINATIONS****NOVEMBER – 2017****15CMA3B****ELECTIVE I : LAPLACE AND FOURIER TRANSFORMS****Time: 3 Hrs****Max. Marks: 75****SECTION – A (10 X 2 =20)****Answer ALL the questions.**

1. Define Laplace Transform.
2. Find  $L[e^{-4t}]$ .
3. Find  $L[\sinh t]$
4. Define Periodic function.
5. Find  $L^{-1} \left[ \frac{6}{(s+2)^4} \right]$ .
6. Find  $L^{-1} \left[ \frac{1}{s^5} \right]$ .
7. Define Fourier sine & cosine integral.
8. Define Fourier transform.
9. Define Convolution theorem.
10. Define parseval's identity.

**SECTION – B (5 X 5 =25)****Answer any FIVE of the following questions.**

11. State & Prove shifting property.
12. Find (i)  $L[\cos^3 t]$  (ii)  $L \left[ \frac{e^{3t} - e^{-2t}}{t} \right]$ .
13. Find  $L^{-1} \left[ \frac{s^2 + 9s + 2}{(s-1)^2(s+2)} \right]$ .
14. Using Laplace transform solve the equation.

$$\frac{d^2y}{dt^2} + 4 \frac{dy}{dt} + 13y = 2e^{-t} \text{ given } y(0) = 0, y'(0) = -1.$$

15. Find the Fourier transform of  $f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$ .
16. Find the Fourier sine transform of  $f(x) = \begin{cases} \sin x & 0 < x < a \\ 0 & x > a \end{cases}$
17. Show that  $\int_0^\infty \frac{\cos sx}{1+s^2} ds = e^{-x}, x \geq 0$ .
18. Using Parseval's identity to show that

$$\int_0^\infty \frac{1}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{2ab(a+b)}$$

**SECTION – C (3 X 10 =30)**

Answer ALL the questions.

19. a) (i) Find  $L[\sin^2 t \cos^3 t]$  (ii)  $L[e^{7t} \cos^2 t]$

(Or)

b) find  $L^{-1} \left[ \frac{s^2}{(s^2+a^2)^2} \right]$ .

20.a) Find  $L^{-1} \left[ \frac{7s^2+23s+30}{(s-2)(s^2+2s+5)} \right]$

(Or)

b) Using Laplace transform  $\frac{dx}{dt} = 2x - 3y, \frac{dy}{dt} = y - 2x$  given that  $x(0) = 8, y(0) = 3$ .

21.a) Find the Fourier transform of  $f(x) = \begin{cases} 1 - x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$  and hence evaluate

$$\int_0^{\infty} \left( \frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$$

(Or)

b) Find the Fourier transform of  $f(x) = \begin{cases} 1 - |x| & |x| < 1 \\ 0 & |x| > 1 \end{cases}$  and hence evaluate

$$\int_0^{\infty} \left( \frac{\sin t}{t} \right)^4 dt.$$

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