Reg No: | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER -2018 **15CMA5A**

LINEAR ALGEBRA

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Time : 3 Hrs **Max.Marks : 75**

SECTION-A (10 x 2 =20)

Answer ALL the questions.

1. *Define a vector space.*
2. *Let S be a non-empty subset of a vector space V. Define the linear span L(S).*
3. *If* $dim_F V=13$ *then what is the* $dim_F Hom(V,V)$*?*
4. *Define an inner product space.*
5. *Define the characteristic root of* $T\epsilon A(V)$.
6. *When a linear transformation is said to be regular?*
7. *Let V be the vector space of all polynomials over F of degree 3 or less ,and let D be the differentiation operator defined by* $(a_0 + a_1x + a_2x^2 + a_3x^3)D = a_1 + 2a_2x + 3a_3x^2$ *.Find the matrix of D with respect to the basis* $\{1,x,x^2,x^3\}$.
8. *Define similar linear transformation and similar matrices.*
9. *Define trace and transpose of a matrix.*
10. *Prove that if A is invertible then* $det(A)\neq 0$ *and* $detA^{-1}=(detA)^{-1}$.

SECTION-B (5 x 5 =25)

Answer any FIVE of the following questions.

11. *Define the Kernel of a Vector Space homomorphism. Showthat it is a subspace.*
12. *If* $v_1, v_2 \dots v_n$ *are in V then either they are linearly independent or some* v_k *is a linear combination of preceding vectors* $v_1, v_2 \dots v_{k-1}$.
13. *Derive Schwarz's inequality.*
14. *If V is a finite dimensional vector space over F then for* $S,T\epsilon A(V)$ *prove that (i)* $r(ST)\leq r(T)$ *(ii)* $r(TS)\leq r(T)$.
15. *If* $\lambda\epsilon F$ *is a characteristic root of* $T\epsilon A(V)$ *then for anypolynomial* $q(x)\epsilon F[x]$*.Prove that* $q(\lambda)$ *is a characteristicroot of* $q(T)$.
16. *Compute the following matrix product:*

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & -1 & -1 \end{bmatrix}$$

17. *If V is n-dimensional over F and if* $T\epsilon A(V)$ *hasall its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.*
18. *State and Prove Jacobson's lemma.*

SECTION-C (3 x 10 =30)

Answer ALL the questions.

19. (a)*If V is a finite dimensional vector space over F and W is subspace of V , then prove that W is finite dimensional over F and dim W ≤ dim V and dim V/W = dim V – dim W.*

(Or)

(b)*If V and W are vector spaces of dimensions m and n respectively then prove that Hom(V,W) is of dimension mn.*

20. (a)*If V is a finite dimensional inner product space, then Prove that V has a orthonormal set as a basis.*

(Or)

(b)*If V is finite dimensional over F, then prove that T∈A(V) is invertible if and only if the constant term of the minimal polynomial for T is not zero.*

21. (a) *If T∈A(V) has all its characteristic roots in F,then prove that there is a basis of V in which the matrix of T is triangular.*

(Or)

(b)*If T has all its characteristic roots in F and $tr(T^i) = 0$ then T is nilpotent.*

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