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**D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**

**SEMESTER EXAMINATIONS**

 **NOVEMBER - 2018 15CMA5A LINEAR ALGEBRA**

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SECTION-A (10 x 2 =20)

 **Answer ALL the questions.**

1. *Define a vector space.*
2. *Let S be a non-empty subset of a vector space V. Define the linear span L(S).*
3. *If dimFV=13 then what is the dimFHom(V,V)?*
4. *Define an inner product space.*
5. *Define the characteristic root of TϵA(V).*
6. *When a linear transformation is said to be regular?*
7. *Let V be the vector space of all polynomials over F of degree 3 or less ,and let D be the differentiation operator defined by*

$(a\_{0}+a\_{1}x+a\_{2}x^{2}+a\_{3}x^{3})D=a\_{1}+2a\_{2}x+3a\_{3}x^{2}$ *.Find the matrix of D with respect to the basis {1,x,*$x^{2}$*,*$x^{3}$*}.*

1. *Define similar linear transformation and similar matrices.*
2. *Define trace and transpose of a matrix.*
3. *Prove that if A is invertible then det(A)≠0 and detA-1=(detA)-1.*

SECTION-B (5 x 5 =25)

 **Answer any FIVE of the following questions.**

1. *Define the Kernel of a Vector Space homomorphism. Show that it is a subspace.*
2. *If* $v\_{1, } v\_{2}… v\_{n} $ *are in V then either they are linearly independent or some* $v\_{k}$ *is a linear combination of preceding vectors* $v\_{1}$*,* $v\_{2}$ *…* $v\_{k-1}$*.*
3. *Derive Schwarz’s inequality.*
4. *If V is a finite dimensional vector space over F then for S,TϵA(V) prove that (i) r(ST)≤r(T)*

*(ii) r(TS)≤r(T).*

1. *If λϵF is a characteristic root of TϵA(V) then for any polynomial q(x)ϵ F[x].Prove that q(λ) is a characteristic root of q(T).*
2. *Compute the following matrix product:*

$$\left[\begin{matrix}1&2&3\\1&-1&2\\3&4&5\end{matrix}\right]\left[\begin{matrix}1&0&1\\0&2&3\\-1&-1&-1\end{matrix}\right]$$

1. *If V is n-dimensional over F and if TϵA(V) has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.*
2. *State and Prove Jacobson’s lemma.*

SECTION-C (3 x 10 =30)

 **Answer ALL the questions.**

1. *(a)* *If V is a finite dimensional vector space over F and W is subspace of V , then prove that W is finite*

 *dimensional over F and dim W ≤ dim V and dim V/W = dim V – dim W.*

*(Or)*

*(b) If V and W are vector spaces of dimensions m and n respectively then prove that Hom(V,W) is of*

 *dimension mn.*

1. *(a)* *If V is a finite dimensional inner product space, then Prove that V has a orthonormal set as a basis.*

*(Or)*

*(b) If V is finite dimensional over F, then prove that TϵA(V) is invertible if and only if the constant*

 *term of the minimal polynomial for T is not zero.*

1. *(a) If TϵA(V) has all its characteristic roots in F, then prove that there is a basis of V in which the*

 *matrix of T is triangular.*

*(Or)*

*(b) If T has all its characteristic roots in F and tr(T i) = 0 then T is nilpotent.*

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