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**D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**

**SEMESTER EXAMINATIONS**

**NOVEMBER - 2018 15CMA5A LINEAR ALGEBRA**

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SECTION-A (10 x 2 =20)

**Answer ALL the questions.**

1. *Define a vector space.*
2. *Let S be a non-empty subset of a vector space V. Define the linear span L(S).*
3. *If dimFV=13 then what is the dimFHom(V,V)?*
4. *Define an inner product space.*
5. *Define the characteristic root of TϵA(V).*
6. *When a linear transformation is said to be regular?*
7. *Let V be the vector space of all polynomials over F of degree 3 or less ,and let D be the differentiation operator defined by*

*.Find the matrix of D with respect to the basis {1,x,,}.*

1. *Define similar linear transformation and similar matrices.*
2. *Define trace and transpose of a matrix.*
3. *Prove that if A is invertible then det(A)≠0 and detA-1=(detA)-1.*

SECTION-B (5 x 5 =25)

**Answer any FIVE of the following questions.**

1. *Define the Kernel of a Vector Space homomorphism. Show that it is a subspace.*
2. *If are in V then either they are linearly independent or some is a linear combination of preceding vectors , … .*
3. *Derive Schwarz’s inequality.*
4. *If V is a finite dimensional vector space over F then for S,TϵA(V) prove that (i) r(ST)≤r(T)*

*(ii) r(TS)≤r(T).*

1. *If λϵF is a characteristic root of TϵA(V) then for any polynomial q(x)ϵ F[x].Prove that q(λ) is a characteristic root of q(T).*
2. *Compute the following matrix product:*
3. *If V is n-dimensional over F and if TϵA(V) has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.*
4. *State and Prove Jacobson’s lemma.*

SECTION-C (3 x 10 =30)

**Answer ALL the questions.**

1. *(a)* *If V is a finite dimensional vector space over F and W is subspace of V , then prove that W is finite*

*dimensional over F and dim W ≤ dim V and dim V/W = dim V – dim W.*

*(Or)*

*(b) If V and W are vector spaces of dimensions m and n respectively then prove that Hom(V,W) is of*

*dimension mn.*

1. *(a)* *If V is a finite dimensional inner product space, then Prove that V has a orthonormal set as a basis.*

*(Or)*

*(b) If V is finite dimensional over F, then prove that TϵA(V) is invertible if and only if the constant*

*term of the minimal polynomial for T is not zero.*

1. *(a) If TϵA(V) has all its characteristic roots in F, then prove that there is a basis of V in which the*

*matrix of T is triangular.*

*(Or)*

*(b) If T has all its characteristic roots in F and tr(T i) = 0 then T is nilpotent.*

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