Reg. No. | | | | | | | | | | |

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER – 2017 **15CMA5A**

LINEAR ALGEBRA

Time : 3 Hrs **Max. Marks : 75**

SECTION-A (10 x 2 = 20)

Answer ALL questions.

1. *Define Vector space.*
2. *Verify whether the vectors (1, 1, 0), (3, 1, 3) and (4, 3, 3) are linearly independent or not.*
3. *Define inner product space.*
4. *Show that the* $W^{\perp}$ *is a subspace of V.*
5. *Define invertible transformation.*
6. *Define characteristic root.*
7. *Define Basis and dimension of a vector space.*
8. *Define similar transformation and similar matrices.*
9. *Define symmetric matrix.*
10. *If A is invertible, then prove that det A* $\neq 0$ *and det* $(A^{-1}) = (\det A)^{-1}$.

SECTION-B (5 x 5 = 25)

Answer any FIVE of the following questions.

11. *If V is a vector space over F. Then prove that the following*
    (i) $\alpha.0 = 0$ *for* $\alpha \in F$
    (ii) $0.v = 0$ *for* $v \in V$
    (iii) $(-\alpha)v = -(\alpha v)$ *for* $\alpha \in F, v \in V$
12. *Define* $L(s)$ *and prove that the L(S) is subspace of V.*
13. *If* $S, T \in A(V)$ *and S is regular then T and* STS^{-1} *have the same minimum polynomial over F.*
14. *State and prove Cauchy's Schwarz inequality.*
15. *If V is finite dimensional over F, then prove that* $T \in A(V)$ *invertible if and only if the constant term of the minimum polynomial is not zero.*
16. $T \in A(V)$ *is invertible if and only if T is onto.*
17. *If V is n – dimensional over F and if* $T \in A(V)$ *all has its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.*
18. *State and prove Jacobson's lemma.*

SECTION-C (3 x 10 = 30)

Answer ALL questions.

19. (a) *If V is a finite dimensional and if W is a subspace of V, then prove that W is finite dimensional, dim W $\leq$ dim V and dim V/W = dim V – dim W.*

(Or)

(b) *If V and W are two vector spaces over F of dimensions m and n respectively, then prove that Hom (V, W) is of dimensions mn over F.*

20. (a) *State and prove Gram-Schmidt orthogonalisation process.*

(Or)

(b) *If A is an algebra, with unit element, over F, then prove that A is isomorphic to a subalgebra of A(V) for some vector space V over F.*

21. (a) *If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.*

(Or)

(b) *T has all its characteristic roots in F and Tr $T^i = 0$ for $i \geq 1$ then prove that T is nilpotent.*

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