Reg.No : | | | | | | | | | | | |

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER - 2018 **15CMA5B**

REAL ANALYSIS - I

Time : 3 Hours **Max. Marks : 75**

SECTION – A (10 x 2 = 20)

Answer ALL the questions.

1. *Prove that the set of all integers is countable.*
2. *Define least upper bound and find l.u.b and g.l.b of* $\{ \frac{1}{2}, \frac{3}{4}, \ldots (2^n-1)/2^n, \ldots \}$.
3. *Define a monotone sequence of real numbers and give example.*
4. *Define convergent sequence.*
5. *Define a monotone sequence.*
6. *Differentiate between absolute and conditional convergence of a series.*
7. *Define a metric space and give an example.*
8. *Prove that* $\lim_{x\to 3} x^2 + 2x = 15$.
9. *Is the sequence* $A = \{ < 0, n/(n+1) > \}^{\infty}_{n=1}$ *convergent in M, where M is the set of all points* $<x, y>$ *in the Euclidean plane* R^2 *such that* $x^2+y^2 < 1$?
10. *Define open ball in real line and in a metric space.*

SECTION – B (5 x 5 = 25)

Answer any FIVE of the following questions.

11. *Prove that inverse image of intersection of two sets is the intersection of the inverse images.*
12. *If A is any nonempty subset of R that is bounded below, then prove that A has a greatest lower bound in R.*
13. *Prove that every convergent sequence is bounded.*
14. *If* $\{s_n\}_{n=1}^{\infty}$ *is convergent to L then prove that* $\{s_n^2\}_{n=1}^{\infty}$ *is convergent to* L^2.
15. *If* $\{s_n\}_{n=1}^{\infty}$*conwages then* $\{s_n\}_{n=1}^{\infty}$ *is a Cauchy sequence. Is the converse true on real line?*
16. *Prove that the series* $\sum^{\infty}_{n=1} (1/n)$ *is divergent.*
17. *Let f be a non-decreasing function on the bounded open interval (a, b). If f is bounded above on (a, b), then* $\lim_{x\to b-} f(x)$ *exists.*
18. *If* f *is continuous at* a *and* g *is continuous at* $f(a)$ *then* gof *is continuous at* a .

SECTION – C (3 x 10 = 30)

Answer ALL the questions.

19. (a) *Prove that the interval [0, 1] is uncountable and hence prove that the set of all real numbers is uncountable.*

(Or)

(b) i) *Prove that a non-decreasing sequence bounded above is convergent.*

ii) *If* $[s_n]$ *is a sequence of non-negative numbers and if* $lim_{n\to\infty} s_n = L$ *then* $L \geq 0$.

20. (a) *Prove that the sequence* $\{ (1 + 1/n)^n \}^{\infty}_{n=1}$ *is convergent.*

(Or)

(b) *If* $\{s_n\}_{n=1}^{\infty}$ *and* $\{t_n\}_{n=1}^{\infty}$ *are sequences of real numbers and if* $lim_{n\to\infty} s_n = L$ *and* $lim_{n\to\infty} t_n = M$

then i) $lim_{n\to\infty} s_n + t_n = L + M$.

ii) $lim_{n\to\infty} s_n t_n = L\, M$.

iii) $lim_{n\to\infty} \frac{s_n}{t_n} = \frac{L}{M}$.

21. (a) *Let <M, ρ> be a metric space and a be a point in M. Let f and g be real-valued functions whose domains are subsets of M. If* $\lim_{x\to a} f(x) = L$ *and* $\lim_{x\to a} g(x) = N$, *the prove that*

i) $lim_{x\to a} f(x) + g(x) = L + N$.

ii) $lim_{x\to a} f(x)g(x) = L\, N$.

(Or)

(b) *The real valued functions* f *is continuous at* $a \in R'$ *if and only if*

$lim_{n\to\infty} x_n = a$ *then* $lim_{n\to\infty} f(x_n) = f(a)$.

* * * * * *