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**D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**

 **SEMESTER EXAMINATIONS**

 **NOVEMBER - 2018 15CMA5B**

 **REAL ANALYSIS - I**

**Time : 3 Hours Max. Marks : 75**

**Section – A (10 x 2 = 20)**

**Answer ALL the questions.**

1. *Prove that the set of all integers is countable.*
2. *Define least upper bound and find l.u.b and g.l.b of { ½, ¾, …(2n-1)/2n,…}.*
3. *Define a monotone sequence of real numbers and give example.*
4. *Define convergent sequence.*
5. *Define a monotone sequence.*
6. *Differentiate between absolute and conditional convergence of a series.*
7. *Define a metric space and give an example.*
8. *Prove that*
9. *Is the sequence A = { < 0, n / (n+1) > }∞n=1 convergent in M, where M is the set of all points <x, y> in the Euclidean plane R2 such that x2+y2 < 1?*
10. *Define open ball in real line and in a metric space.*

**Section – B ( 5 x 5 = 25 )**

**Answer any Five of the following questions.**

1. *Prove that inverse image of intersection of two sets is the intersection of the inverse images.*
2. *If A is any nonempty subset of R that is bounded below, then prove that A has a greatest lower bound*

 *in R.*

1. *Prove that every convergent sequence is bounded.*
2. *If is convergent to L then prove that is convergent to L2.*
3. *If conwages then is a Cauchy sequence. Is the converse true on real line?*
4. *Prove that the series ∑∞n=1 (1/n) is divergent.*
5. *Let f be a non-decreasing function on the bounded open interval (a, b). If f is bounded above on (a, b), then limx→b-f(x) exists.*
6. *If is continuous at and is continuous at then is continuous at .*

**Section – C ( 3 x 10 = 30 )**

**Answer ALL the questions.**

1. *(a)* *Prove that the interval [0, 1] is uncountable and hence prove that the set of all real numbers is*

 *uncountable.*

*(Or)*

*(b) i) Prove that a non-decreasing sequence bounded above is convergent.*

 *ii) If is a sequence of non-negative numbers and if then*

1. *(a) Prove that the sequence { ( 1 + 1/n)n }∞n=1 is convergent.*

*(Or)*

*(b) If and are sequences of real numbers and if and*

 *then i)*

 *ii)*

 *iii)*

1. *(a) Let <M, ρ> be a metric space and a be a point in M. Let f and g be real-valued functions whose*

 *domains are subsets of M. If and , the prove that*

1.

*(Or)*

 *(b) The real valued functions is continuous at if and only if*

 *then*

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