Reg No: | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER - 2017 **15CMA5B**

REAL ANALYSIS - I

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Time : 3 Hrs **Max.Marks : 75**

SECTION-A (10 x 2 =20)

Answer ALL the questions.

1. *For any two real valued functions f and g, what is $max(f,g)$ and $min(f,g)$.*
2. *Define Countable set.*
3. *Define Convergent sequence.*
4. *Define Bounded sequence.*
5. *Define Cauchy sequence.*
6. *If $\sum_{n=1}^{\infty} a_n$ is Convergent series. Show that $lim_{n\to\infty} a_n = 0$.*
7. *When do you say that $f(x)$ approaches L as x approaches infinity?*
8. *Define Metric space.*
9. *Define Continuous function.*
10. *Define the Open ball of radius r about a.*

SECTION-B (5 x 5 =25)

Answer any FIVE of the following questions.

11. *Prove that $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$.*
12. *If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers & if $lim_{n\to\infty} S_n = L$ then $L \geq 0$.*
13. *If $\{S_n\}_{n=1}^{\infty}$ is a convergent sequence then it is bounded.*
14. *Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.*
15. *If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ converges.*
16. *Prove that $\lim_{x\to 3}(x^2 + 2x) = 15$.*
17. *If $lim_{x\to a} f(x) = L$ & $lim_{x\to a} g(x) = M$ then prove that $lim_{x\to a}\big(f(x) + g(x)\big) = L + M$.*
18. *If f and g are real valued functions if f is continuous at a and g is continuous at $f(a)$ then prove that gof is continuous at a.*

SECTION-C (3 x 10 =30)

Answer ALL the questions.

19. (a) *Show that countable union of countable set is countable.*

(Or)

(b) Prove that the set $[0,1] = \{x/0 \leq x \leq 1\}$ *is uncountable.*

20. *(a) Prove that the sequence* $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ *is convergent.*

(Or)

(b) State & prove Nested interval theorem.

21. *(a) Let* $< M, \rho >$ *be a metric space and let* a *be a point in* M*. Let* f *and* g *be real valued functions whose domains are subsets of* M*. If* $lim_{x\to a}\, f(x) = L$ *and* $lim_{x\to a}\, g(x) = N$ *then* $lim_{x\to a}\, f(x)g(x) = LN$.

(Or)

(b) *If* f *is continuous at a if and only if* $\lim_{x\to a} x_n = a$ *then* $\lim_{x\to a} f(x_n) = f(a)$.

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