

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE – 1**SEMESTER EXAMINATIONS****NOVEMBER – 2017****15CMA5E****ELECTIVE III : GRAPH THEORY****Time: 3 Hrs****Max. Marks: 75****SECTION – A (10 X 2 =20)****Answer ALL the questions.**

1. Define complete graph.
2. Prove that $\delta \leq \frac{2q}{p} \leq \Delta$.
3. Define incidence matrix.
4. Define Union.
5. Define connected.
6. Prove that if G is a K-connected graph then $q \geq \frac{pk}{2}$.
7. Define Eulerian trail.
8. Define Hamiltonian cycle.
9. State vizing theorem.
10. Define plane graph.

SECTION – B (5 X 5 =25)**Answer any FIVE of the following questions.**

11. Prove that in any graph G the number of points of odd degree is even.
12. Prove that any self complementary graphs has 4n or 4n+1 points.
13. Define adjacency matrix with an example.
14. Prove that a graph G with p points and $\delta \geq \frac{p-1}{2}$ is connected.
15. Prove that there is no 3-connected graph with 7 edges.
16. Prove that every connected graph has a spanning tree.
17. State and prove Euler theorem.
18. If G is K-critical, then prove that $\delta(G) \geq K - 1$.

SECTION – C (3 X 10 =30)**Answer ALL the questions.**

19. a) Show that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4} \right]$.

(Or)

- b) State and prove Eula's theorem on connected plane graph.

20. a) Let G_1 be a (p_1, q_1) graph and G_2 a (p_2, q_2) graph prove that

- (i) $G_1 \cup G_2$ is a $(p_1 + p_2, q_1 + q_2)$ graph
- (ii) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$ graph
- (iii) $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph.

(Or)

b) Prove that a graph G with at least two points is bipartite if all its cycles are of even length.

21. a) State and prove Chavatal theorem.

(Or)

b) Show that K_5 is non – planar.

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