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D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER - 2017 15CPMA1A

ALGEBRA - I

Time : 3 Hrs **Max.Marks : 75**

SECTION-A (5x 6 =30)

Answer ALL the questions.

1. *(a) Prove that N(a) is a subgroup of G.*

   (Or)

   (b) State and prove second part of Sylow's theorem.

2. *(a) Prove that the intersection of any number of submodules of an R-module M is a submodule of M.*

   (Or)

   (b) Prove that G is solvable if and only if $G^{(k)} = (e)$ for some integer k.

3. *(a) lf V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then T satisfies a polynomial of degree n over F.*

   (Or)

   (b) Prove that if $u \in V_1$ is such that $uT^{n_1-k} = 0$, where $0 < k \leq n_1$, then $u = u_0T^k$ for some $u_0 \in V_1$.

4. *(a) Prove: Suppose that $V = V_1 \oplus V_2$, where V_1 and V_2 are subspaces of V invariant under T. Let T_1 and T_2 be the linear transformations induced by T on V_1 and V_2, respectively. If the minimal polynomial of T_1 over F is $p_1(x)$ while that of T_2 is $p_2(x)$, then the minimal polynomial for T over F is the least common multiple of $p_1(x)$ and $p_2(x)$.*

   (Or)

   (b) Let $T \in A_f(V)$ have all its distinct characteristic roots, $\lambda_1, \lambda_2, \ldots, \lambda_k$, in F. Then a basis of V can be found in which the matrix T is of the form $\begin{pmatrix} J_1 & & & \\ & J_2 & & \\ & & \ddots & \\ & & & J_k \end{pmatrix}$ where each

   $J_1 = \begin{pmatrix} B_{i1} & & & \\ & B_{i2} & & \\ & & \ddots & \\ & & & B_{ir_i} \end{pmatrix}$ *and where $B_{i1}, \ldots, B_{ir_i}$ are basic Jordan blocks belonging to λ_i.*

5. *(a) If F is of characteristic 0 and if S and T, in $A_F(V)$, are such that $ST - TS$ commutes with S, then $ST - TS$ is nilpotent.*

   (Or)

   (b) If λ is a characteristic root of the normal transformation N and if $vN = \lambda v$ then $vN^* = \bar{\lambda} v$.

SECTION-B (3x15 =45)

Answer any THREE of the following questions.

6. *State and prove Cauchy theorem.*

7. *a) Suppose that G is the internal direct product of* $N_1, \ldots, N_n$. *Then show that for* $i \neq j$,

   $N_i \cap N_j = (e)$, *and if* $a \in N_i$, $b \in N_j$ *then* $ab = ba$. (10)

   b) Show that S_n *is not solvable for* $n \geq 5$. (5)

8. *Show that there exists a subspace W of V, invariant under T, such that* $V = V_1 \oplus W$.

9. *Prove that elements S and T in* $A_F(V)$ *are similar in* $A_F(V)$ *if and only if they have the same elementary divisors.*

10. *a) If* $T \in A(V)$ *then show that the following:*

    i) $T^* \in A(V)$;

    ii) $(T^*)^* = T$;

    iii) $(S + T)^* = S^* + T^*$;

    iv) $(ST)^* = T^*S^*$; *for all* $S, T \in A(V)$ *and all* $\lambda \in F$. (10)

    b) Prove that the normal transformation N is Hermitian if and only if its characteristic roots are real.

    (5)

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