Reg.No. | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER – 2018 15CPMA1A

ALGEBRA - I

Time : 3 Hrs **Max. Marks : 75**

SECTION-A (5x6=30)

Answer ALL the questions.

1. *(a) If $O(G) = p^n$ where p is a prime number, prove that $Z(G) \neq \{e\}$.*

(Or)

(b) If G is a finite group, p a prime and $p^n | O(G)$ but $p^{n+1} \nmid O(G)$, prove that any two subgroups of G of order p^n are conjugate.

2. *(a) Show that G is solvable if and only if $G^{(k)} = (e)$ for some integer k.*

(Or)

(b) Let G be a group and suppose that G is the internal direct product of $N_1, N_2, \cdots N_n$ and let $T = N_1 \times N_2 \times \cdots \times N_n$. Show that G and T are isomorphic.

3. *(a) If $V = V_1 \oplus V_2 \oplus \cdots \oplus V_k$ where each subspace V_i is of dimension n_i and is invariant under $T \in A(V)$, prove that a basis of V can be found so that the matrix of T in this basis of the form*

$$\begin{pmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & A_k \end{pmatrix}$$

(Or)

(b) Let $T \in A(V)$ is nilpotent. Prove that $\alpha_0 + \alpha_1 T + \cdots + \alpha_m T^m$ where the $\alpha_i \in F$ is intertible if $\alpha_0 \neq 0$.

4. *(a) Let the two matrices A, B in F_n are similar in K_n where K is an extension of F. Prove that A and B are already similar in F_n.*

(Or)

(b) (i) Suppose that $V = V_1 \oplus V_2$ where $V_1 \& V_2$ are subspaces of V invariant under T. Prove that the minimal polynomial for T over F is is the least common multiple of $p_1(x)$ and $p_2(x)$.

(ii) Define companion matrix of $f(x)$ in $F(x)$.

5. *(a) If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, prove that $T = 0$.*

(Or)

(b) If F is a field of characteristic 0 and if $T \in A(V)$ is such that $tr(T^i) = 0$ for all $i \leq 1$, prove that T is nilpotent.

SECTION-B (3x15 =45)

Answer any THREE of the following questions.

6. *If p is a prime number and $p^{\alpha}|O(G)$, prove that G has a subgroup of order p^{α}.*

7. *Prove that every finite abelian group is the direct product of cyclic groups.*

8. *If $T \in A(V)$ has all its characteristic roots in F then there is a basis of V in which the matrix of T is triangular.*

9. *Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.*

10. *(a) Show that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.*

    (b) If F is of characteristic 0 and if S and T, in $A_F(V)$ are such that ST – TS commutes with S, prove that ST – TS is nilpotent.

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