Reg.No :

D. K. M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER – 2017 **15CPMA1B**

REAL ANALYSIS - I

Time : 3 Hours **Max. Marks : 75**

SECTION – A (5 x 6 = 30)

Answer ALL the questions.

1. *(a) If f is continuous on [a,b] and* f' *exists and is bounded in the interior, say* $|f'(x)| \leq A$ *for all x in (a,b) then prove that f is of bounded variation on [a,b].*

(Or)

(b) Let $\sum a_n$ *be a given series with real – valued terms and define*

$p_n = \dfrac{|a_n| + a_n}{2}$, $q_n = \dfrac{|a_n| - a_n}{2}$, *n=1,2,3…. , If* $\sum a_n$ *is conditionally convergent, then prove that both* $\sum p_n$ *and* $\sum q_n$ *diverge.*

2. *(a) Assume that* α *is increasing on [a,b], then prove that if P' is finer than P, we have*

$$U(P', f, \alpha) \leq U(P, f, \alpha) \quad \text{and} \quad L(P', f, \alpha) \geq L(P, f, \alpha).$$

(Or)

(b) Assume that α *is increasing on [a,b]. If* $f \in R(\alpha)$ *on [a,b], then* $|f| \in R(\alpha)$ *on [a,b] then prove that*

$$\left|\int_a^b f(x)d\alpha(x)\right| \leq \int_a^b |f(x)| d\alpha(x).$$

3. *(a) If f is continuous on [a,b] and if* α *is of bounded variation on [a,b], then prove that* $f \in R(\alpha)$ *on [a,b].*

(Or)

(b) State and prove the second fundamental theorem of integral calculus.

4. *(a) Assume that* $\sum_{n=0}^{\infty} a_n$ *converges absolutely and has sum A, and suppose* $\sum_{n=0}^{\infty} b_n$ *converges with sum B. Then prove that Cauchy product of these two series converges and has sum AB.*

(Or)

(b) Assume f and all its derivatives are nonnegative on a compact interval [b, b+r], then prove that if $b \leq x \leq b + r$*, the Taylor's series* $\sum_{k=0}^{\infty} \dfrac{f^k(b)}{k!}(x-b)^k$ *converges to f(x).*

5. (a) *State and prove the Cauchy's condition for uniform convergence.*

(Or)

(b) *Assume that* $\lim_{n\to\infty} f_n = f$ *on [a,b]. If* $g \in R$ *on [a,b]. Define*

$$h(x) = \int_a^x f(t)g(t)dt, \quad h_n(x) = \int_a^x f_n(t)g(t)dt, \text{ if } x \in [a,b], \text{ then prove that } h_n \to h \text{ uniformly on } [a,b].$$

SECTION – B (3 x 15 = 45)

Answer any THREE of the following questions.

6. (i) *Let f be a bounded variation on [a,b] and assume that* $c \in (a,b)$, *then prove that f is bounded variation on [a,c] and on [c,b] and also prove* $V_f(a,b) = V_f(a,c) + V_f(c,b)$.

(ii) *Let* $\sum a_n$ *be an absolutely convergent series having sum s, then prove that every rearrangement of* $\sum a_n$ *also converges absolutely and has sum s.*

7. *If* $f \in R(\alpha)$ *on [a,b], then prove that* $\alpha \in R(f)$ *on [a,b] and also*

$$\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = f(b)\alpha(b) - f(a)\alpha(a).$$

8. *State and prove the Lebesgue's criterion for Riemann - integrability.*
9. *State and prove the Abel's limit theorem.*
10. *Let* α *be of bounded variation on [a,b]. Assume that each term of the sequence* $\{f_n\}$ *is a real valued function such that* $f_n \in R(\alpha)$ *on [a,b] for each n=1,2,3…. Assume that* $f_n \to f$ *uniformly on [a,b] and define* $g_n(x) = \int_a^x f_n(t)d\alpha(t)$ *if* $x \in [a,b], n = 1,2,3,\ldots$ *then prove that*

(i) $f \in R(\alpha)$ *on [a,b].*

(ii) $g_n \to g$ *uniformly on [a,b] where* $g(x) = \int_a^x f(t)d\alpha(t)$.

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