Reg.No : | | | | | | | | | | | |

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER – 2016 **15CPMA1B**

REAL ANALYSIS - I

Time : 3 Hours **Max. Marks : 75**

SECTION – A (5 x 6 = 30)

Answer ALL the questions.

1. (a) Assume that both f and g are of bounded variation on $[a, b]$. Prove that $f . g$ is of bounded variation and $V_{f.\,g} \leq AV_f + BV_g$. Can $\frac{1}{f}$ be of bounded variation? Give explanation.

(Or)

(b) Let $\sum a_n$ be an absolutely convergent series having sum s. Prove that every rearrangement of $\sum a_n$ also converges absolutely and has sum s.

2. (a) Assume that c$\in (a, b)$. If the Riemann - Stieltjes integrals $\int_a^c f\, d\alpha$ and $\int_c^b f\, d\alpha$ exist, prove that $\int_a^b f\, d\alpha$ also exist and $\int_a^b f\, d\alpha = \int_a^c f\, d\alpha + \int_c^b f\, d\alpha$.

(Or)

(b) If $f \in R(\alpha)$ on $[a, b]$, then prove that $\alpha \in R(f)$ on $[a, b]$ and

$$\int_a^b f(x) d\,\alpha(x) + \int_a^b \alpha(x)\, d\, f(x) = f(b)\alpha(b) - f(a)\alpha(a).$$

3. (a) If f is continuous on $[a, b]$ and if α is of bounded variation on $[a, b]$, then prove that $f \in R(\alpha)$ on $[a, b]$.

(Or)

(b) State and prove the second mean value theorem for Riemann integrals.

4. (a) State and prove Tauber's theorem.

(Or)

(b) Prove that the infinite product $\pi\, u_n$ converges if and only if for every $\varepsilon > 0$, there exists an N suchthat $n > N$ implies $|u_{n+1}.\, u_{n+2} \dots u_{n+k} - 1| < \varepsilon\ for\ k = 1, 2, \dots$

5. (a) Let $\{f_n\}$ be a sequence of functions defined on a set S. Prove that there exists a function f suchthat $f_n \longrightarrow f$ uniformly on S if and only if, the following condition is satisfied:
For every $\varepsilon > 0$, there exists an N suchthat $m > N$ and $n > N$ implies $|f_m(x) - f_n(x)| < \varepsilon$, for every x in S.

(Or)

(b) Give an example of a sequence of functions f_n on $[0, 1]$ suchthat $\{f_n\}$ converges in the mean but $\{f_n(x)\}$ does not converge at any point x in $[0, 1]$.

SECTION – B (3 x 15 = 45)

Answer any THREE of the following questions.

6. *Let* f *be continuous on* $[a,b]$. *Prove that* f *is of bounded variation on* $[a,b]$ *if and only if* f *can be expressed as the difference of two increasing continuous functions.*

7. *If* $\alpha \nearrow$ *on* $[a,b]$, *prove that the following statements are equivalent.*
   i. $f \in R(\alpha)$ *on* $[a,b]$.
   ii. f *satisfies Riemann's condition with respect to* α *on* $[a,b]$.
   iii. $\underline{I}(f,\alpha) = \bar{I}(f,\alpha)$.

8. *Let* f *be defined and bounded on* $[a,b]$ *and let* D *devote the set of discontinuities of* f *in* $[a,b]$. *Prove that* $f \in R$ *on* $[a,b]$ *if and only if* D *has measure zero.*

9. *a) State and prove Merten's theorem.* *(10Marks)*

   b) If a series is convergent with sum S, *prove that it is also* $(C,1)$ *summable with Cesaro sum* S. *(5Marks)*

10. *a) Assume that* $f_n \longrightarrow f$ *uniformly on a set S. If each* f_n *is continuous at a point c of* S, *then prove that the limit function* f *is also continuous at c.* *(5Marks)*

    b) State and prove the Dirichlet's test for uniform convergence of the series $\sum f_n(x).g_n(x)$. *(10Marks)*

******