Reg No: | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER - 2017 **15CPMA1C**

ORDINARY DIFFERENTITAL EQUATIONS

Time : 3 Hrs **Max.Marks : 75**

SECTION-A (5x 6 =30)

Answer ALL the questions.

1. (a) For any real x_0 and constants α, β, prove that there exists an solution φ of the initial value problem $L(y) = 0, y(x_0) = \alpha, y'(x_0) = \beta$ on $-\infty < x < \infty$.

   (Or)

   (b) Solve the non – homogeneous equations $y" - y' - 2y = e^{-x}$ using variation of parameters.

2. (a) State and prove that Existence theorem for n^{th} order differential equation.

   (Or)

   (b) Find the complete solution of $y''' + y'' + y' + y = 1, \psi(0) = 0, \psi'(0) = 1, \psi''(0) = 0$.

3. (a) Prove that there exists n linearly independent solutions of $L(y) = 0$ on I.

   (Or)

   (b) Find all the solutions of the equation $y'' - \frac{2}{x^2}y = x, 0 < x < \infty$.

4. (a) Find the indicial polynomial of the equation $L(y) = x^2y'' + \frac{3}{2}xy' + xy = 0$.

   (Or)

   (b) Prove that the equation $x^2y'' + a(x)xy' + b(x)y = 0$, where a, b have power series expansions which are convergent for $|x| < r_0,\ r_0 > 0$.

5. (a) Find the solution of $y' = xy, y(0) = 1$ using successive approximation.

   (Or)

   (b) Test whether $f(x, y) = xy^2$ on $R: |x| \leq 1, |y| \leq 1$ satisfies Lipschitz condition.

SECTION-B (3x15 =45)

Answer any THREE of the following questions.

6. Prove that two solutions φ_1, φ_2 of $L(y) = 0$ are linearly independent on an interval I if and only if $W(\varphi_1, \varphi_2)(x) \neq 0$ for all x in I.
7. Consider the equation with constant coefficients $L(y) = P(x)e^{ax}$, where P is the polynomial given by $P(x) = b_0x^m + b_1x^{m-1} + \cdots + b_m,\ b_0 \neq 0$. Suppose a is a root of the characteristic polynomial p of L of multiplicity j. Then prove that there is a unique solution ψ of $L(y) = P(x)e^{ax}$ of the form $\psi(x) = x^j(C_0x^m + C_1x^{m-1} + \cdots + C_m)e^{ax}$, where $C_0, C_1, \cdots C_m$ are constants determined by the annihilator method.
8. Let $\varphi_1, \cdots, \varphi_n$ be n solutions of $L(y) = 0$ on an interval I and let x_0 be any point in I. Then prove that $W(\varphi_1, \cdots, \varphi_n)(x) = exp\left[-\int_{x_0}^{x} a_1(t)dt\right] W(\varphi_1, \cdots, \varphi_n)x_0$.
9. Solve the Bessel equation $xy'' + xy' + (x^2 - \alpha^2)y = 0$.
10. State and prove the Existence theorem.

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