Reg No: | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER - 2017 **15CPMA1E**

ELECTIVE : GRAPH THEORY

Time : 3 Hrs **Max.Marks : 75**

SECTION-A (5x 6 =30)

Answer ALL the questions.

1. *(a) Define the degree* $d_G(v)$ *of a vertex* v *in* G. *Prove that* $\sum_{v \in V} d_G(v) = 2\varepsilon$ *and hence deduce that in any graph, the number of vertices of odd degree is even.*

   (Or)

   (b) If G *is a tree, then show that* $\varepsilon = \upsilon - 1$

2. *(a) Define connectivity and edge connectivity. Show that* $\kappa \leq \kappa' \leq \delta$.

   (Or)

   (b) If G *is a graph with* $\upsilon \geq 3$ *and* $\delta \geq \frac{\upsilon}{2}$ *then* G *is Hamiltonian.*

3. *(a) If* G *is a* $k-$*regular bipartite graph with* $k > 0$, *then* G *has a perfect matching.*

   (Or)

   (b) Let G *be a connected graph that is not an odd cycle. Then* G *has a 2- edge colouring in which both colours are represented at each vertex of degree at least two.*

4. *(a) Prove that* $r(k,l) \leq \binom{k+l-2}{k-1}$

   (Or)

   (b) Let G *be a* $k-$*critical graph with a 2- vertex cut* $\{u,v\}$. *Then* $d(u) + d(v) \geq 3k - 5$

5. *(a) Use Jordan curve theorem to show that* K_5 *is nonplanar.*

   (Or)

   (b) State and prove Euler's formula for connected plane graph.

SECTION-B (3x15 =45)

Answer any THREE of the following questions.

6. *Define bipartite graph. Prove that a graph is bipartite if and only if it contains no odd cycles.*
7. *Let* G *be a simple graph with degree sequence* $(d_1, d_2, ...d_\upsilon)$, *where* $d_1 \leq d_2 \leq ... \leq d_\upsilon$ *and* $\upsilon \geq 3$. *Suppose that there is no value of* $m < \frac{\upsilon}{2}$ *for which* $d_m \leq m$ *and* $d_{\upsilon - m} < \upsilon - m$. *Then G is Hamiltonian.*
8. *State and prove Vizing's theorem.*
9. *State and prove Brook's theorem.*
10. *Prove that the following three statements are equivalent:*
    - *(i) every planar graph is 4- vertex- colourable;*
    - *(ii) every plane graph is 4- face- colourable;*
    - *(iii) every simple 2- edge- connected 3- regular planar graph is 3- edge- colourable.*

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