Reg No: | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER- 2018 15CPMA1E ELECTIVE : GRAPH THEORY

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Time: 3 Hrs Max.Marks : 75

SECTION-A (5x 6 =30)

Answer ALL the questions.

1. *(a)If G is a tree then prove that $\varepsilon = \nu - 1$.*

   (Or)

   (b)In any graph, the number of vertices of odd degree is even.

2. *(a)Prove that a nonempty connected graph is Eulerian if and only if it has no vertices of odd degree.*

   (Or)

   (b) Show that if G is simple and 3-regular, then $K = K'$.

3. *(a) Show that a tree has at most one perfect matching.*

   (Or)

   (b) If G is bipartite, then $\chi' = \Delta$.

4. *(a) A subset S of V is an independent set of G if and only if V\S is a covering of G.*

   (Or)

   (b) If G is k -critical, then $\delta \geq k-1$.

5. *(a)If G is a simple planar graph, with $\gamma \geq 3$, then $\in \leq 3\nu - 6$.*

   (Or)

   (b) Without using Euler's formula, prove that complete graph K_5 is non-planar.

SECTION-B (3x15 =45)

Answer any THREE of the following questions.

6. *(i) Prove that a graph is bipartite if and only if it contains no odd cycle.* *(10)*

   (ii) Prove that every non-trivial tree has at least two vertices of degree one. *(5)*

7. *(i) If G is a simple graph with $\gamma \geq 3$ and $\delta \geq \gamma / 2$ then prove that G is Hamiltonian.* *(10)*

   (ii)With usual notations prove that $k \leq k' \leq \delta$. *(5)*

8. *Let G be a bipartite graph with bipartition (X, Y) . Then prove that G contains a matching that saturates every vertex of X if and only if $N(S) \geq |S|$ for all $S \subseteq X$. Hence prove that a regular bipartite graph has a perfect matching.* *(12+3)*

9. *In usual notations, prove that if $\delta > 0$ then prove that $\alpha' + \beta' = \nu$.*

10. *(i) State and prove Euler's formula for plane graphs.* *(10)*

    (ii) Prove that every planar graph is five vertex colourable. *(5)*

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