

**D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**  
**SEMESTER EXAMINATIONS**  
**NOVEMBER – 2016**  
**ELECTIVE : GRAPH THEORY**

**15CPMA1E**

**Time : 3 Hours**

**Max. Marks : 75**

**SECTION – A (5 x 6 = 30)**

**Answer ALL the questions.**

1. (a) Prove that the number of points of odd degree is even in any graph  $G$ .  
(Or)  
(b) Prove that a connected graph is a tree if and only if every edge is a cut edge.
2. (a) A graph  $G$  with  $v \geq 3$  is 2 - connected if and only if any two vertices of  $G$  are connected by at least two internally disjoint paths. Prove.  
(Or)  
(b) Let  $G$  be a simple graph with degree sequence  $(d_1, d_2, \dots, d_n)$  where  $d_1 \leq d_2 \leq \dots \leq d_n$  and  $n \geq 3$ . Suppose that there is no value of  $m$  less than  $n/2$  for which  $d_m \leq m$  and  $d_{n-m} \leq n - m$  then prove that  $G$  is Hamiltonian.
3. (a) Prove that a matching  $M$  in  $G$  is a maximum matching if and only if  $G$  contains no  $M$  - augmenting path.  
(Or)  
(b) State and prove Hall's theorem.
4. (a) Using the standard notations, show that  $\alpha + \beta = \gamma$ .  
(Or)  
(b) If  $G$  is  $k$  - critical, then show that  $\delta \geq k - 1$ .
5. (a) Prove that a graph  $G$  is embeddable in the plane if and only if it is embeddable on the sphere.  
(Or)  
(b) If  $G$  is a simple planar graph with  $\gamma \geq 3$  vertices, then prove that  $\epsilon \geq 3\gamma - 6$ , and hence show that  $K_5$  is non-planar.

**SECTION – B (3 x 15 = 45)**

**Answer any THREE of the following questions.**

6. Prove that a graph is bipartite if and only if it contains no odd cycle.
7. Prove that a connected graph has an Euler trail, if and only if it has at most two vertices of odd degree.
8. State and prove Vizing's theorem.
9. a) If  $G$  is simple then prove that  $\pi_k(G) = \pi_k(G - e) - \pi_k(G, e)$  for any edge  $e$  of  $G$ .  
b) Find Chromatic polynomial for the graph  $K_{1,3}$ . (9+6 Marks)
10. State and prove five colour theorem.

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