Reg.No. | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

APRIL – 2016 **15CPMA2B**

REAL ANALYSIS - II

Time : 3 Hrs **Max. Marks : 75**

SECTION-A (5 x 6 = 30)

Answer ALL questions.

1. *(a) If f and g are in $L(I)$, then so are the functions $f^+, f^-, |f|$, max (f, g) and min (f, g) moreover we have*

$$\left|\int_I f\right| \leq \int_I |f|.$$

(Or)

(b) Let f be defined on the half infinite integral $I = (a, \infty)$. Assume that f is Lebesque integrable on the compact interval $[a, b]$ for each $b \geq a$ and that there is a positive constant M such that $\int_a^b |f| \leq M$ for all $b \geq a$. Then $f \in L(I)$, the $\lim_{b\to\infty} \int_a^b f$ exists and $\int_a^{+\infty} f = \lim_{b\to\infty} \int_a^b f$.

2. *(a) Let f be defined on I and assume that $\{f_n\}$ is a sequence of measurable functions on I such that $f_n(x) \to f(x)$ almost every where on I. Then f is measurable on I.*

(Or)

(b) Let $\{f_n\}$ be a Cauchy sequence of complex valued function in $L^2(I)$ that is assume that for every $\in > 0$ there is an integer N such that $\|f_m - f_n\| < \in$ whenever $m \geq n \geq N$, then there exists a function f in $L^2(I)$ such that

$$\lim_{n\to\infty} \| f_n - f \| = 0.$$

3. (a) Let $\{\varphi_0, \varphi_1, \cdots\cdots\}$ be orthonormal on I, and assume that $f \in L^2(I)$. Define two sequence of functions $\{s_n\}$ and $\{t_n\}$ on I as follows $s_n(x) = \sum_{k=0}^{n} c_k \varphi_k(x)$, $t_n(x) = \sum_{k=0}^{n} b_k \varphi_k(x)$ where $c_k = (f, \varphi_k)$ for $k = 0,1,2, \cdots\cdots$ and $b_0, b_1, b_2, \cdots\cdots\cdots$ are arbitrary complex number. Then for each n we have $\|f - s_n\| \leq \|f - t_n\|$ more over, equality holds if and only if $b_k = c_k$ for $k = 0,1,2, \cdots\cdots n$.

(Or)

(b) Let f be real valued and continuous function on a compact interval $[a, b]$. Then for $\in > 0$ there is a polynomial p(which may depend on $\in$) such that

$$|f(x) - p(x)| < \in \; for \; x \; in \; [a, b].$$

4. (a) Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b, with total derivative f'(b). Then the composite function $h = f \circ g$ is differentiable at a, and the total derivative $h'(a)$ is given by $h'(a) = f'(b) \circ g'(a)$ the composition of the linear functions $f'(b) and \; g'(a)$.

(Or)

(b) State and derive Taylor's formula.

5. (a) Let $B = B(a:r)$ be an n – ball in R^n, let ∂B denotes it boundary, $\partial B = \{x: \|x - a\| = r\}$ and let $\bar{B} = B \cup \partial B$ denotes its closure. Let $f = \{f_1, f_2, \cdots \cdots f_n\}$ be continuous on $\bar{B}$ and assume that all the partial derivatives$D_j f_i(x)$ exists if $x \in B$. Assume further that $f(x) \neq f(a)$ $if x \in \partial B$ and that the Jacobian determinant $J_f(x) \neq 0$ for each x in B. Then $f(B)$, the image of B under f contains an n - ball with centre at $f(a)$.

(Or)

(b) Let A be an open subset of R^n and assume that f from $A \to R^n$ is continuous and has finite partial derivatives $D_j f_i$ on A. If f is one – one on A and if $J_{f(x)} \neq 0$ for each x in A, then $f(A)$ is open.

SECTION-B (3 x 15 = 45)

Answer any THREE of the following questions.

6. Let $\{f_n\}$ be sequence of Lebesque integrable functions on an interval I. Assume that

(a) Sequence $\{f_n\}$ converges almost every where on I to limit function f, and

(b) There is non negative function g in L(I) such that for all $n \geq 1, |f_n(x)| \leq g(x)$ alomost every where on I.

Then the limit function $f \in L(I)$, the sequence $\left\{\int_I f_n\right\}$ converges and $\int_I f = \lim_{n\to\infty} \int_I f_n$.

7. *Let* $\{g_n\}$ *be sequence of functions in* $L^2(I)$ *such that the series* $\sum_{n=1}^{\infty}\|g_n\|$ *converges. Then the series of functions* $\sum_{n=1}^{\infty} g_n$ *converges almost every where on I to a function g in* $L^2(I)$*, and we have* $\|g\| = \lim_{n\to\infty}\|\sum_{k=1}^{n} g_k\| \leq \sum_{k=1}^{\infty}\|g_k\|$.

8. *State and prove Riesz – Fischer theorem.*

9. *Assume that one of the partial derivatives* $D_1 f, D_2 f, \cdots\cdots\cdots D_n f$ *exists at c and that the remaining n -1 partial derivatives exist in some n-ball B(c) and are continuous at c. Then f is differentiable at c.*

10. *State and prove Inverse function theorem.*

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