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**D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**  
**SEMESTER EXAMINATIONS**  
**APRIL – 2016**                      **15CPMA2D**  
**Tensor Analysis and Relativity**

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Time : 3 Hours

Max. Marks : 75

**SECTION – A (5 x 6 = 30)**

Answer ALL the questions.

1. (a) State and prove any five properties of Kronecker delta  $\delta_j^i$ .

(Or)

(b) If  $A^i$  and  $B^j$  are two contravariant vectors, then prove that their outer product  $A^i B^j$  is a tensor of type (2, 0). Give an example of a (2, 0) tensor that cannot be expressed as the outer product of two contravariant vectors.

2. (a) Prove that the fundamental metric tensor  $g_{ij}$  is symmetric.

(Or)

(b) Define Christoffel symbols of both kinds. Prove that the maximum number of independent components of the Christoffel symbols in an  $n$ -dimensional Riemannian space is  $\frac{n^2(n+1)}{2}$ .

3. (a) If  $a_{ij}$  is a symmetric non-singular tensor ( $|a_{ij}| \neq 0$ ) of type (0, 2), such that  $a_{ij, k} = 0$ , then prove that

$$\left\{ \begin{matrix} l \\ i \ j \end{matrix} \right\} = \frac{1}{2} a^{lk} \left( \frac{\partial a_{ik}}{\partial x^l} + \frac{\partial a_{jk}}{\partial x^l} - \frac{\partial a_{ij}}{\partial x^k} \right).$$

(Or)

(b) Define Riemann-Christoffel curvature tensor and show that it is skew-symmetric with respect to last two covariant indices.

4. (a) Explain the principle of relativity.

(Or)

(b) Write a note on time dialation.

5. (a) Discuss the idea of four - vector to describe the separation of two events occurring in space time.

(Or)

(b) Derive the transformation equations for momentum and energy.

### SECTION – B ( 3 x 15 = 45 )

Answer any THREE of the following questions.

6. A covariant vector has components  $xy$ ,  $2y - z^2$ ,  $xz$  in rectangular Cartesian coordinates. Determine its components in spherical polar coordinates.

7. Evaluate Christoffel symbols for Riemannian space where  $g_{ij} = 0$  if  $i \neq j$ .

8. In  $V_2$ , show that the components of Ricci tensor are proportional to the components of the metric tensor.

9. a) Suppose that relative to a frame  $S$ , a particle has a velocity

$$\bar{u} = u_x \hat{i} + u_y \hat{j} + u_z \hat{k} \text{ where } u_x = \frac{dx}{dt} \text{ etc. Find the velocity of this}$$

particle as measured in the frame of reference  $S^1$  moving with a velocity  $v_x$  relative to  $S$ .

b) Write a note on Length contraction.

10. Explain the principle of equivalence of mass and energy in special relativity theory.

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