

**D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**  
**SEMESTER EXAMINATIONS**  
**NOVEMBER – 2017**  
**COMPLEX ANALYSIS - I**

15CPMA3A

Time : 3 Hours

Max. Marks : 75

**SECTION – A (5 x 6 = 30)**

Answer ALL the questions.

1. (a) State and prove the Cauchy's estimate.

(Or)

(b) Compute  $\int_{|z|=1} e^z z^{-n} dz$ .2. (a) Find the residues of  $f(z) = \frac{e^z}{(z-a)^2}$ .

(Or)

(b) State and prove the argument principle.

3. (a) If  $u$  is harmonic, then  $f(z) = \frac{\partial u}{\partial x} - i \frac{\partial u}{\partial y}$  is analytic in  $\Omega$ .

(Or)

(b) State and prove the maximum principle for harmonic functions.

4. (a) State and prove the Hurwitz theorem.

(Or)

(b) Prove that  $\pi \cot \pi z = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{2z}{z^2 - n^2}$ 5. (a) Show that  $\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$ .

(Or)

(b) Prove Every function which is meromorphic in the whole plane is the quotient of two entire functions.

**SECTION – B (3 x 15 = 45)**

Answer any THREE of the following questions.

6. a) State and prove Morera's theorem.

(6)

b) State and prove the maximum principle.

(9)

7. State and prove residue theorem.

8. Prove that  $\int_0^{\pi} \frac{d\theta}{a + \cos \theta} = \frac{\pi}{\sqrt{a^2 - 1}}$ ;  $a > 1$ .

9. State and prove Weierstrass theorem.

10. Prove that  $\sqrt{\pi} \Gamma(2z) = 2^{2z-1} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right)$ .

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