Reg.No :

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE – 1

SEMESTER EXAMINATIONS

NOVEMBER – 2017 15CPMA3B

DISCRETE MATHEMATICS

Time: 3 Hrs **Max. Marks: 75**

SECTION – A (5 X 6 =30)

Answer ALL the questions.

1 a) Prove that the cardinality of a finite Boolean algebra is always of the form 2^n and any two Boolean algebras with the same cardinality are isomorphic.

(Or)

b) Let B be a Boolean algebra. Then prove that an ideal (filter) M in B is maximal if and only if for any $b \in B \quad either \quad b \in M \quad or \quad b' \in M,$ but not both, hold.

2 a) Draw the diagram for the switching circuit $p = x_1(x_2(x_3 + x_4) + x_3(x_5 + x_6))$

(Or)

b) Draw the symbolic representation of $p = (x^1{}_1 x_2)^1 + x_3$

3 a) Let F be a finite field of characteristic p. Then prove that F contains p^n elements, where n=[F:Z_p]

(Or)

b)Let p be a prime and let m,n be natural numbers, then prove that

(i) If F_p^m is a subfield of F_p^n then m|n.

(ii)If m|n then $F_p^m \mapsto F_p^n$. There is exactly one subfield of F_p^n with p^m elements.

4 a)Let f be an irreducible polynomial over F_q of degree k. f divides $x^{q^n} - x$ if and only if k divides n.

(Or)

b) Let $f \in F_q[x]$ be an polynomial of degree $m \geq 1$ with then prove that there exists a positive integer $e \leq q^m - 1$ such that f divides $x^e - 1$

5 a) Prove that (i) $\sum_{a \in F_q} X(a) = 0$ for a nontrivial character of F_q.

(ii) The characters of F_q form a group which is isomorphic to the group (F_q,+).

(iii) For any $a, b \in F_q, \quad a \neq -b,$ we have $\sum X(a)X(b) = 0$ where the summation runs through all character of F_q.

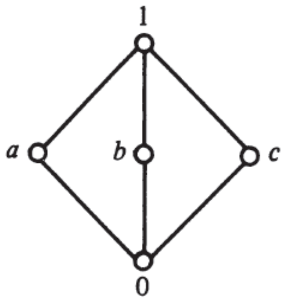
(Or)

b) Prove that a linear code $C \subseteq V_n$ is cyclic if and only if C is a principal ideal in V_n, generated by $g \in C$

SECTION – B (3 X 15 =45)

Answer any THREE of the following questions.

6. Prove that a moular lattice is distributive if and only if none its sublattices is isomorphic to the "diamond lattice" V_3^5 , whose Hasse diagram is given below.



7. In a large room there are electrical switches next to the three doors to operate the central lighting. The three switches operate alternatively. Find the contact diagram, disjunctive normal form and symbolic representation of the problem.

8. State and prove Mobius inversion formula of Additive form and multiplicative form.

9. State and prove Chinese remainder theorem for Polynomials.

10. Let C be a linear (n,k) code over F_q and $C^{\perp}$ its dual code. If A(X,Y) is the weight enumerator of C and $A^{\perp}(X,Y)$ is the weight of enumerator of $C^{\perp}$, then prove that

$$A^{\perp}(X,Y) = \frac{1}{q^k} A(Y - X, Y + (q-1)X)$$

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