Reg.No. | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER – 2018 **15CPMA3B**

DISCRETE MATHEMATICS

Time : 3 Hrs **Max. Marks : 75**

SECTION-A (5x6=30)

Answer ALL the questions.

1. *(a) Let V be a lattice then Prove that the following implication hold by*

   (i) (V is a Boolean algebra) $\Rightarrow$ (V is relatively complemented)

   (ii) (V is relatively complemented) $\Rightarrow$ (V is sectionally complemented)

   (iii) (V is finite and sectionally complemented) $\Rightarrow$ (every $0 \neq a, a \in V$ is a join of finitely many atoms).

   (Or)

   (b) Let B be a Boolean algebra. Then prove that the set $P_n(B)$ is a Boolean algebra and a subalgebra of the Boolean algebra $F_n(B)$ of all functions from B^n into B.

2. *(a) Draw the diagram for the switching circuit $p = x_1(x_2(x_3 + x_4) + x_3(x_5 + x_6))$*

   (Or)

   (b) Draw the symbolic representation of $p = (x^1{}_1 x_2)^1 + x_3$

3. *(a) Let F be a finite field of characteristic p. Then prove that F contains p^n elements, where $n=[F:Z_p]$.*

   (Or)

   (b) Let p be a prime and let m,n be natural numbers, then prove that

   (i) If F_p^m is a subfield of F_p^n then m|n.

   (ii) If m|n then $F_p^m \mapsto F_p^n$. There is exactly one subfield of F_p^n with p^m elements.

4. *(a) Let f be an irreducible polynomial over F_q of degree k. Prove that f divides $x^{q^n} - x$ if and only if k divides n.*

   (Or)

   (b) Let $f \in F_q[x]$ be an polynomial of degree $m \geq 1$ then prove that there exists a positive integer $e \leq q^m - 1$ such that f divides $x^e - 1$.

5. (a) *Prove that*

(i) $\sum_{a \in F_q} X(a) = 0$ *for a nontrivial character of* F_q.

(ii) *(ii)The characters of* F_q *form a group which is isomorphic to the group* $(F_q,+)$.

(iii)*For any* $a,b \in F_q$, $a \neq -b$, *we have* $\sum X(a)X(b) = 0$ *where the summation runs through all character of* F_q.

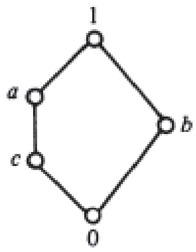
(Or)

(b) *Prove that a linear code* $C \subseteq V_n$ *is cyclic if and only if C is a principal ideal in* V_n, *generated by* $g \in C$

SECTION-B (3x15 =45)

Answer any THREE of the following questions.

6. *Prove that a lattice L is Modular if and only if none its sublattices is isomorphic to the "Pentagon lattice"* V_4^5 *, whose Hasse diagram is given below.*



7. *In a large room there are electrical switches next to the three doors to operate the central lighting. The three switches operate alternatively. Find the contact diagram, disjunctive normal form and symbolic representation of the problem.*

8. *Define Mobius function . State and prove Mobius inverstion formula of Additive form and multiplicative form.*

9. *State and prove Chinese remainder theorem for Polynomials.*

10. *Let C be a linear (n,k) code over* F_q *and* $C^{\perp}$ *its dual code. If A(X,Y) is the weight enumerator of C and* $A^{\perp}(X,Y)$ *is the weight of enumerator of* $C^{\perp}$, *then prove that* $A^{\perp}(X,Y) = \frac{1}{q^k} A(Y-X, Y+(q-1)X)$

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