

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**SEMESTER EXAMINATIONS****NOVEMBER - 2017****15CPMA3C****TOPOLOGY****Time : 3 Hrs****Max.Marks : 75****SECTION-A (5x 6 =30)****Answer ALL the questions.**

1. (a) Prove that the lower limit topology τ' on \mathbb{R} is strictly finer than the standard topology τ .

(Or)

- (b) If A is a subspace of X and B is a subspace of Y , then prove that product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

2. (a) State and prove the pasting lemma.

(Or)

- (b) State and prove sequence lemma.

3. (a) Prove that the image of a connected space under a continuous map is connected.

(Or)

- (b) State and prove intermediate value theorem.

4. (a) Prove that every compact subset of Hausdorff space is closed.

(Or)

- (b) State and prove maximum and minimum value theorem.

5. (a) Suppose that X has a countable basis. Then prove that every open covering of X contains a countable sub collection covering X .

(Or)

- (b) Prove that every metrizable space is normal.

SECTION-B (3x15 =45)**Answer any THREE of the following questions.**

6. (i) Let Y be a subspace of X ; let A be a subset of Y ; let \bar{A} denote the closure of A in X . Then prove that the closure A in Y equals $A \cap Y$.

- (ii) Let A be a subset of the topological space X ; let A' be the set of all limit points of A . Then prove that $\bar{A} = A \cup A'$.

7. Prove that the topologies on \mathbb{R}^n induced by the Euclidean metric d and the square metric ρ are the same as the product topology on \mathbb{R}^n .

8. Prove that the Cartesian product of connected spaces is connected.

9. Prove that the product of finitely many compact spaces is compact.

10. State and prove Urysohn lemma.

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