

**D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1****SEMESTER EXAMINATIONS****NOVEMBER - 2017****15CPMA3C****TOPOLOGY****Time : 3 Hrs****Max.Marks : 75****SECTION-A (5x 6 =30)****Answer ALL the questions.**

1. (a) Prove that the lower limit topology  $\tau'$  on  $R$  is strictly finer than the standard topology  $\tau$ .

(Or)

- (b) If  $A$  is a subspace of  $X$  and  $B$  is a subspace of  $Y$ , then prove that product topology on  $A \times B$  is the same as the topology  $A \times B$  inherits as a subspace of  $X \times Y$ .

2. (a) State and prove the pasting lemma.

(Or)

- (b) State and prove sequence lemma.

3. (a) Prove that the image of a connected space under a continuous map is connected.

(Or)

- (b) State and prove intermediate value theorem.

4. (a) Prove that every compact subset of Hausdorff space is closed.

(Or)

- (b) State and prove maximum and minimum value theorem.

5. (a) Suppose that  $X$  has a countable basis. Then prove that every open covering of  $X$  contains a countable sub collection covering  $X$ .

(Or)

- (b) Prove that every metrizable space is normal.

**SECTION-B (3x15 =45)****Answer any THREE of the following questions.**

6. (i) Let  $Y$  be a subspace of  $X$ ; let  $A$  be a subset of  $Y$ ; let  $\bar{A}$  denote the closure of  $A$  in  $X$ . Then prove that the closure  $A$  in  $Y$  equals  $A \cap Y$ .
- (ii) Let  $A$  be a subset of the topological space  $X$ ; let  $A'$  be the set of all limits points of  $A$ . Then prove that  $\bar{A} = A \cup A'$ .
7. Prove that the topologies on  $R^n$  induced by the Euclidean metric  $d$  and the square metric  $\rho$  are the same as the product topology on  $R^n$ .
8. Prove that the Cartesian product of connected spaces is connected.
9. Prove that the product of finitely many compact spaces is compact.
10. State and prove Urysohn lemma.

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