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D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER - 2016 **15CPMA3C**

TOPOLOGY

Time : 3 Hrs **Max.Marks : 75**

SECTION-A (5x 6 =30)

Answer ALL the questions.

1. (a) Let $\mathbb{B}$ and $\mathbb{B}'$ be bases for the topologies τ and τ' respectively on X. Then prove that the following are equivalent.
   i) τ is finer than τ' .
   ii) for each $x \in X$ and each basis element $B \in \mathbb{B}$ containing x, there is a basis element $B \in \mathbb{B}'$ such that $x \in B' \subset B$.

   (Or)

   (b) Let Y be a subspace of X. Then prove that a set A is closed if and only if it equals the intersection of a closed set of X with Y.

2. (a) State and prove the pasting lemma.

   (Or)

   (b) State and prove the sequence lemma.

3. (a) Prove that the image of a connected space under a continuous map is connected.

   (Or)

   (b) State and prove the intermediate value theorem.

4. (a) Prove every closed subspace of a compact space is compact.

   (Or)

   (b) Let $f: X \to Y$ be a continuous map of the compact metric space (X, d_X) to the metric (Y, d_Y). Then prove that f is uniformly continuous.

5. (a) Prove that a subspace of a regular space is regular and a product of regular spaces is regular.

   (Or)

   (b) Show that every compact Hausdroff space is normal.

SECTION-B (3x15 =45)

Answer any THREE of the following questions.

6. Let A be a subset of the topological space X. Then
   (a) $x \in \bar{A}$ if and only if every open set U containing x intersects A
   (b) Supposing the topology of X is given by a basis, then $x \in \bar{A}$ if and only if every basis element B containing x intersects A.
7. Let $\bar{d}(a, b) = min\{|a - b|, 1\}$ be the standard bounded Metric on R. If x and y are 2 points. of R^w, define $D(x, y) = sup\left\{\frac{\bar{d}(x_i, y_i)}{i}\right\}$ then prove that D is a metric that induces the product topology on R^w.
8. If L is a linear continuum in the order topology, then prove that L is connected and so are intervals and rays in L.
9. State and prove Lebesgue number lemma.
10. State and prove Urysohn's lemma.

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