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D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

NOVEMBER - 2017 **15CPMA3D**

PROBABILITY THEORY

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Time: 3 Hrs **Max.Marks : 75**

SECTION-A (5x 6 =30)

Answer ALL the questions.

1. (a) *The probabilities of X, Y and Z becoming managers are* $\frac{4}{9}, \frac{2}{9}$ *and* $\frac{1}{3}$ *respectively. The probabilities that the Bonus scheme will be introduced if X, Y and Z become managers are* $\frac{3}{10}, \frac{1}{2}$ *and* $\frac{4}{5}$ *respectively.*

   i) *What is the probability that Bonus will be introduced, and*
   ii) *If the bonus scheme has been introduced, what is the probability that the Manager appointed was X?*

(Or)

(b) *If X and Y are two independent random variables such that* $f(x) = e^{-x}, x \geq 0$ *and* $f(x) = e^{-y}, y \geq 0$, *find the probability distribution of* $U=\frac{X}{X+Y}$ *and V=X+Y. Are U and V independent?*

2. (a) *State and Prove Chebyshev's inequality.*

(Or)

(b) *Prove that the correlation coefficient* ρ *between any two random variables X and Y satisfies the double inequality* $-1 \leq \rho \leq 1$.

3. (a) *Prove that if a random variable* $X = X_1 + \cdots + Xn$ *is a sum of independent random variables* $X1, \ldots, Xn$ *then* $\varphi(X(t)) = \varphi(X_1(t)) \times \cdots \times \varphi(Xn(t))$.

(Or)

(b) *Find the density function of the random variable whose characteristic function is* $\varphi(t) = e^{-t^2/2}, -\infty < t < \infty$.

4. (a) *Let* X_n *be a binomial distribution with parameters n and p. If* $\lambda = np$. *Prove that* $\lim_{n\to\infty} P(X_n = r) = \frac{\lambda^r}{r!}e^{-\lambda}$.

(Or)

(b) *Obtain the mean and variance of a Beta distribution.*

5. (a) *Let* $\{X_k\}$ *be a sequence of independent random variables and let the variance* $D^2(X_k)$ *of* X_k *exists. If* $\sum_{k=1}^{\infty}\frac{D^2(X_k)}{k^2} < \infty$, *prove that the sequence* $\{X_k\}$ *obeys the strong law of large numbers with* $C_n = \frac{1}{n}\sum_{k=1}^{n} E(X_k)$.

(Or)

(b) *State and prove Khintchine Weak law of large numbers.*

SECTION-B (3x15 =45)

Answer any THREE of the following questions.

6. *Find the marginal density, conditional distribution with the joint density function* $f(x,y) = Ae^{-(x+y)}, 0 \leq x \leq \infty, 0 \leq y \leq \infty$. *Check whether X and Y are independent.*

7. *Let (X_1, X_2, ……, X_n) be a n-dimensional random variable. Obtain the regression hyper plane of second type of the random variable X_1 on the remaining variables X_2, X_3, ….X_n. Also find the regression curves of the first type of the random variable X on Y for the joint density function* $f(x,y) = \frac{1}{3}(x+y)$ *where* $0 \leq x \leq 1, 0 \leq y \leq 2$.

8. *State Gnedenko result and prove by an example.*

9. *Define Cauchy distribution and Laplace distribution and derive their characteristic functions.*

10. *State and prove the Lindeberg-Levy central limit theorem.*

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