

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE – 1**SEMESTER EXAMINATIONS****APRIL – 2016****CAMA2A / CAMA4A****ALLIED : MATHEMATICS - II****Time: 3 Hrs****Max. Marks: 75****SECTION – A (10 X 2 =20)****Answer ALL the questions.**

1. Evaluate : $\int_0^{\frac{\pi}{2}} \sin^7 x dx$.
2. Evaluate : $\int x^3 \cos x dx$.
3. Evaluate : $\int_0^1 \int_0^2 xy^2 dy dx$.
4. Define the Fourier series of $f(x)$.
5. Eliminate the arbitrary constants a and b from $z = (x - a)^2 + (y - b)^2 + 1$.
6. Solve : $pq = 1$.
7. Find $L(\sin^2 2t)$.
8. Find $L^{-1}\left(\frac{1}{(s+a)^2}\right)$
9. If $\phi(x, y, z) = x^2 y - 2y^2 z^3$ find $\nabla\phi$, at the point $(1, -1, 2)$.
10. State Green's theorem in the plane.

SECTION – B (5 X 5 =25)**Answer any FIVE of the following questions.**

11. Show that $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$.
12. Find the mass centre of the first quadrant area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, using double integral.
13. Express $f(x) = \frac{x}{2} (-\pi < x < \pi)$ as a Fourier series and hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
14. Solve : $p^2 + q^2 = x + y$.
15. Solve : $z^2(p^2 + q^2 + 1) = 1$.
16. Evaluate: $L^{-1}\left(\frac{s+2}{(s^2+4s+5)^2}\right)$.
17. Find the directional derivative of $xy + y^2 - z^2$ in the direction of the vector $3\vec{l} - 2\vec{j} + \vec{k}$ at $(1, 2, 1)$.
18. If $\vec{F} = 3xy\vec{l} - y^3\vec{j}$, compute $\int_c \vec{F} \cdot d\vec{r}$ along $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.

SECTION – C (3 X 10 =30)**Answer ALL the questions.**

19. a) If $I_{m,n} = \int_0^{\pi/2} \cos^m x \cos nx dx$ prove that $I_{m,n} = \frac{m}{m+1} I_{m-1, n-1}$ Hence prove that $I_{m,n} = \frac{\pi}{2^{m+1}}$
(Or)
b) Find the volume common to the cylinders $x^2 + y^2 = a^2, x^2 + z^2 = a^2$.
20. a) Determine the Fourier series expansion of the function $f(x) = x \sin x$ in the interval $(0, 2\pi)$
(Or)
b) Solve: $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$.
21. a) Using Laplace transform, Solve $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 5y = e^{-2t}$, given that $y = 0, \frac{dy}{dt} = 1$, when $t = 0$.
(Or)
b) Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{l} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

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