Reg. No. | | | | | | | | |

**D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**

**SEMESTER EXAMINATIONS**

**NOVEMBER – 2016** **CMA5B**

# REAL ANALYSIS

**Time : 3 Hrs** **Max. Marks : 75**

## SECTION-A (10 x 2 = 20)

**Answer ALL questions.**

1. *Define countable and uncountable set.*
2. *Define least upper bound of a set.*
3. *When we say a sequence* $\{s_n\}_{n=1}^{\infty}$ *of real number oscillates.*
4. *Define n^th partial sum of a series.*
5. *Prove that* $\lim_{x\to 3} x^2 + 2x = 15$.
6. *When we say a function from a metric space to another metric space is continuous?*
7. *Define open set.*
8. *Define derivative of f at a point* $c$.
9. *Write down the Maclaurin series for a function* $f(x)$.
10. *Explain the word 'almost everywhere'.*

## SECTION-B (5 x 5 = 25)

**Answer any FIVE of the following questions.**

11. *Prove that the image of the union of two sets is the union of their images.*
12. *If A is any non – empty sub set of R. If A is of bounded below then prove that A has a greatest lower bound in R.*
13. *If the sequence of real numbers* $\{s_n\}_{n=1}^{\infty}$ *is convergent. Prove that the above sequence is bounded.*
14. *If* $f$ *and* $g$ *are real valued functions. If* $f$ *is continuous at* $a$ *and* $g$ *is continuous at* $f(a)$. *Then prove that* $g \circ f$ *is convergent at* $a$.
15. *If* $\lim_{x\to a} f(x) = L$ *and* $\lim_{x\to a} g(x) = M$ *then prove that* $\lim_{x\to a}[f(x) + g(x)] = L + M$.
16. *Prove that differentiability implies continuity and prove that the converse is not true.*
17. *State and prove Rolle's Theorem.*
18. *If* $f \in \mathbb{R}[a,b]$ *then prove that* $|f| \in \mathbb{R}[a,b]$ *and* $\left|\int_a^b f\right| \le \int_a^b |f|$.

## SECTION-C (3 x 10 = 30)

**Answer ALL questions.**

19. *(a) Prove that countable union of countable sets is countable.*

*(Or)*

*(b) Prove the sequence* $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ *is convergent.*

20. *(a) Let* $(M, \rho)$ *be a metric space and let a be a point in M. Let f and g be real valued functions whose domains are subsets of M. If* $\lim_{x\to a} f(x) = L$ *and* $\lim_{x\to a} g(x) = N$ *then prove that* $\lim_{x\to a} f(x)g(x) = LN$.

*(Or)*

*(b) State and prove chain rule for derivatives.*

21. *(a) If* $f \in \mathbb{R}[a, b]$ *and* $a < c < b$ *then prove that*

i. $f \in \mathbb{R}[a, c]$ *and* $f \in \mathbb{R}[c, b]$

ii. $\int_a^b f = \int_a^c f + \int_c^b f$.

*(Or)*

*(b) State and prove Taylor's formula for integral form of remainder.*

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